Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.1-a+b-x+c-x^2-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [143]. This is test number [32].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (143)	% 0. (0)
Mathematica	% 100. (143)	% 0. (0)
Maple	% 79.02 (113)	% 20.98 (30)
Maxima	% 65.03 (93)	% 34.97 (50)
Fricas	% 79.02 (113)	% 20.98 (30)
Sympy	% 32.87 (47)	% 67.13 (96)
Giac	% 72.73 (104)	% 27.27 (39)

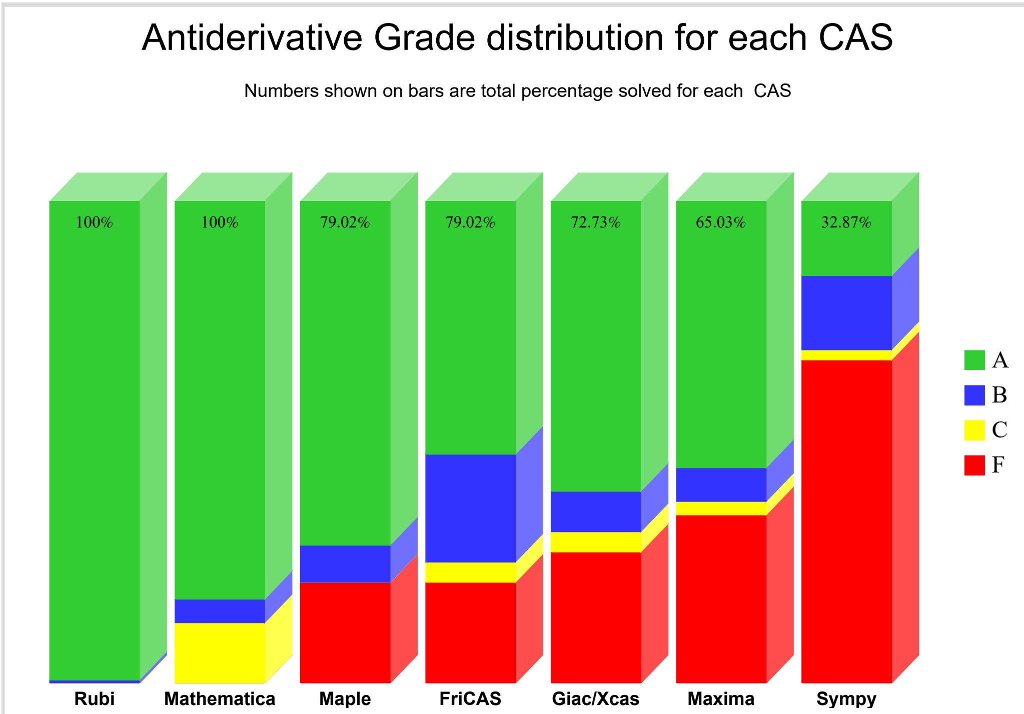
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

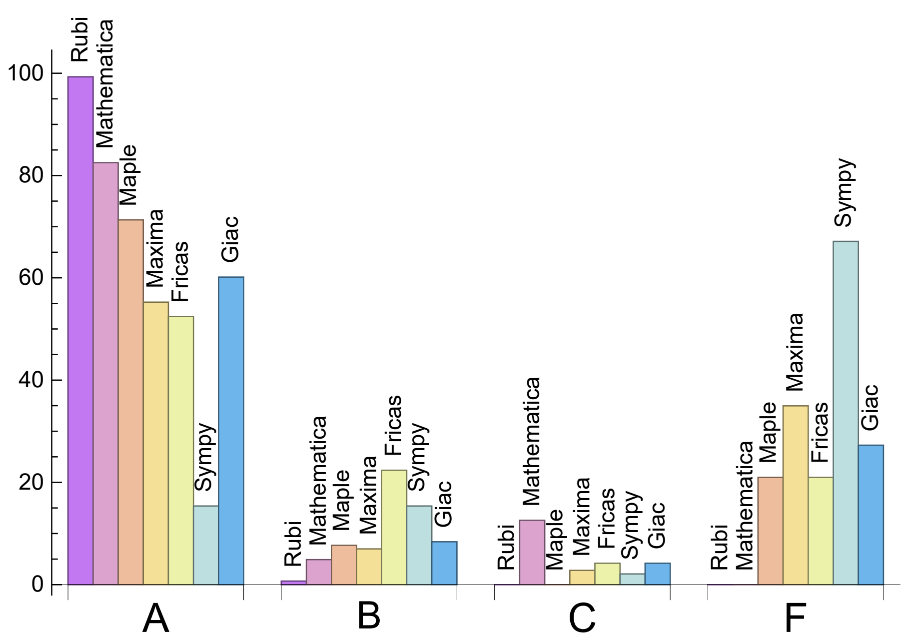
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.3	0.7	0.	0.
Mathematica	82.52	4.9	12.59	0.
Maple	71.33	7.69	0.	20.98
Maxima	55.24	6.99	2.8	34.97
Fricas	52.45	22.38	4.2	20.98
Sympy	15.38	15.38	2.1	67.13
Giac	60.14	8.39	4.2	27.27

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.07	83.41	1.02	38.	1.
Mathematica	0.02	45.29	1.03	40.	1.
Maple	0.07	56.69	1.15	32.	0.86
Maxima	1.57	63.89	1.48	46.	1.41
Fricas	2.15	161.85	4.	115.	3.22
Sympy	0.77	118.6	2.39	70.	1.81
Giac	1.3	63.22	1.51	46.	1.21

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{14, 15, 102\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

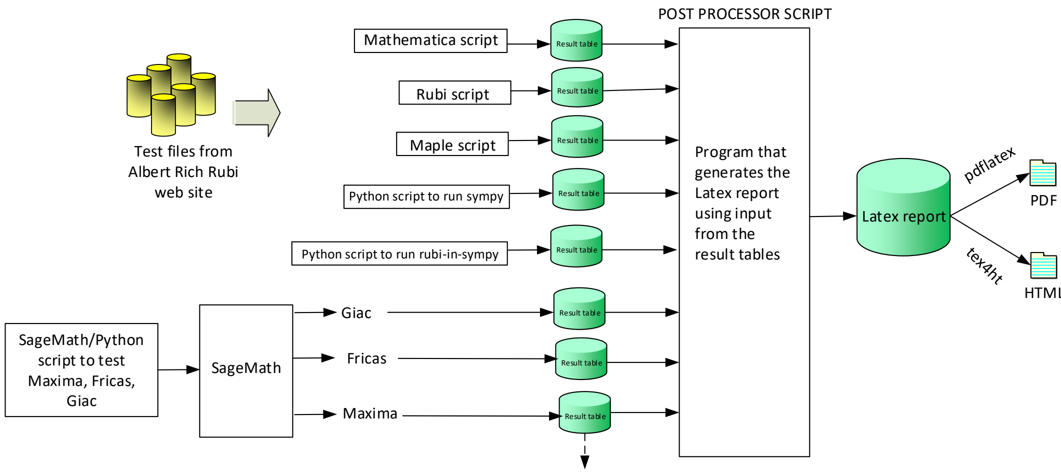
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { 83 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { 17, 21, 25, 28, 29, 83, 102 }

C grade: { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 128, 129, 130, 131, 138 }

B grade: { 25, 74, 75, 76, 77, 83, 101, 102, 125, 126, 127 }

C grade: { }

F grade: { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.1.4 Maxima

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 49, 50, 51, 52, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 78, 79, 80, 81, 82, 84, 85, 86, 87, 91, 92, 93, 94, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 128, 129, 130, 131

B grade: { 17, 74, 75, 76, 77, 83, 98, 99, 100, 101 }

C grade: { 71, 73, 111, 122 }

F grade: { 1, 25, 26, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 88, 89, 90, 95, 96, 97, 125, 126, 127, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 22, 23, 24, 26, 28, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 78, 79, 81, 82, 86, 88, 89, 91, 92, 93, 94, 98, 101, 102, 103, 105, 106, 107, 108, 109, 110, 112, 113, 116, 119, 121, 123, 130, 131, 138 }

B grade: { 17, 18, 21, 25, 27, 74, 75, 76, 77, 80, 83, 84, 85, 87, 90, 95, 96, 97, 99, 100, 104, 114, 115, 117, 118, 120, 124, 125, 126, 127, 128, 129 }

C grade: { 70, 71, 72, 73, 111, 122 }

F grade: { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.1.6 Sympy

A grade: { 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 68, 69, 78, 79, 80, 81, 82, 91, 92, 93, 94, 138 }

B grade: { 53, 54, 61, 62, 63, 74, 75, 76, 77, 83, 84, 85, 87, 88, 89, 90, 95, 96, 97, 98, 99, 100 }

C grade: { 86, 101, 102 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 64, 65, 66, 67, 70, 71, 72, 73, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.1.7 Giac

A grade: { 1, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 27, 28, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 68, 69, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 115, 116, 118, 119, 120, 121, 123, 124, 128, 129, 130, 131, 138 }

B grade: { 64, 74, 75, 76, 77, 83, 101, 114, 117, 125, 126, 127 }

C grade: { 70, 71, 72, 73, 111, 122 }

F grade: { 2, 3, 4, 5, 17, 18, 19, 20, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 67, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	142	173	0	626	0	178
normalized size	1	1.	0.97	1.18	0.	4.26	0.	1.21
time (sec)	N/A	0.054	0.199	0.05	0.	2.288	0.	1.25

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	119	91	176	320	0	0
normalized size	1	1.	0.98	0.75	1.45	2.64	0.	0.
time (sec)	N/A	0.031	0.089	0.104	1.897	2.283	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	88	71	139	246	0	0
normalized size	1	1.	0.93	0.75	1.46	2.59	0.	0.
time (sec)	N/A	0.022	0.087	0.094	1.785	2.279	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	76	51	103	184	0	0
normalized size	1	1.	1.1	0.74	1.49	2.67	0.	0.
time (sec)	N/A	0.015	0.07	0.095	1.711	2.29	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	64	31	66	131	0	0
normalized size	1	1.	1.49	0.72	1.53	3.05	0.	0.
time (sec)	N/A	0.01	0.052	0.094	1.79	2.116	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	88	82	158	247	0	77
normalized size	1	1.	0.87	0.81	1.56	2.45	0.	0.76
time (sec)	N/A	0.028	0.072	0.045	1.725	2.24	0.	1.301

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	64	122	189	0	63
normalized size	1	1.	0.99	0.81	1.54	2.39	0.	0.8
time (sec)	N/A	0.019	0.053	0.044	1.714	2.074	0.	1.294

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	68	46	85	143	0	50
normalized size	1	1.	1.19	0.81	1.49	2.51	0.	0.88
time (sec)	N/A	0.013	0.051	0.044	1.824	2.187	0.	1.355

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	58	28	49	101	0	36
normalized size	1	1.	1.66	0.8	1.4	2.89	0.	1.03
time (sec)	N/A	0.009	0.035	0.045	1.758	2.181	0.	1.306

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	28	49	82	0	34
normalized size	1	1.	0.91	0.8	1.4	2.34	0.	0.97
time (sec)	N/A	0.01	0.043	0.056	1.79	2.17	0.	1.289

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	58	28	49	105	0	36
normalized size	1	1.	1.66	0.8	1.4	3.	0.	1.03
time (sec)	N/A	0.01	0.036	0.044	1.776	2.075	0.	1.228

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	44	42	74	111	0	47
normalized size	1	1.	0.86	0.82	1.45	2.18	0.	0.92
time (sec)	N/A	0.01	0.067	0.052	2.576	2.095	0.	1.374

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	40	33	55	85	0	45
normalized size	1	1.	1.14	0.94	1.57	2.43	0.	1.29
time (sec)	N/A	0.007	0.03	0.052	1.129	2.162	0.	1.257

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	37	37	48	33	58	85	0	45
normalized size	1	1.	1.3	0.89	1.57	2.3	0.	1.22
time (sec)	N/A	0.007	0.038	0.051	2.326	2.351	0.	1.285

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	39	39	46	33	58	90	0	50
normalized size	1	1.	1.18	0.85	1.49	2.31	0.	1.28
time (sec)	N/A	0.007	0.029	0.049	1.229	2.174	0.	1.327

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	70	75	150	223	0	100
normalized size	1	1.	0.84	0.9	1.81	2.69	0.	1.2
time (sec)	N/A	0.018	0.025	0.051	1.3	2.226	0.	1.302

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	53	10	28	62	0	0
normalized size	1	1.	3.31	0.62	1.75	3.88	0.	0.
time (sec)	N/A	0.006	0.017	0.105	1.796	2.476	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	38	100	0	0
normalized size	1	1.	0.92	0.81	1.46	3.85	0.	0.
time (sec)	N/A	0.003	0.005	0.123	1.222	2.646	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	36	42	74	178	0	0
normalized size	1	1.	0.68	0.79	1.4	3.36	0.	0.
time (sec)	N/A	0.007	0.011	0.098	1.208	2.512	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	48	62	111	288	0	0
normalized size	1	1.	0.61	0.78	1.41	3.65	0.	0.
time (sec)	N/A	0.012	0.015	0.108	1.379	2.413	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	40	9	11	47	0	11
normalized size	1	1.	3.33	0.75	0.92	3.92	0.	0.92
time (sec)	N/A	0.006	0.014	0.048	1.787	2.331	0.	1.221

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	25	38	66	0	39
normalized size	1	1.	0.95	1.14	1.73	3.	0.	1.77
time (sec)	N/A	0.002	0.006	0.047	1.168	2.165	0.	1.322

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	31	35	74	119	0	53
normalized size	1	1.	0.69	0.78	1.64	2.64	0.	1.18
time (sec)	N/A	0.006	0.01	0.048	1.236	2.197	0.	1.355

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	45	111	180	0	66
normalized size	1	1.	0.76	0.67	1.66	2.69	0.	0.99
time (sec)	N/A	0.012	0.015	0.048	1.363	2.247	0.	1.627

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	47	35	0	55	0	20
normalized size	1	1.	3.92	2.92	0.	4.58	0.	1.67
time (sec)	N/A	0.008	0.014	0.05	0.	2.232	0.	2.638

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	45	37	0	59	0	49
normalized size	1	1.	1.88	1.54	0.	2.46	0.	2.04
time (sec)	N/A	0.008	0.02	0.057	0.	2.158	0.	1.806

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	14	7	11	42	0	8
normalized size	1	1.	1.4	0.7	1.1	4.2	0.	0.8
time (sec)	N/A	0.006	0.01	0.065	3.884	2.276	0.	1.617

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	33	14	23	43	0	24
normalized size	1	1.	2.06	0.88	1.44	2.69	0.	1.5
time (sec)	N/A	0.004	0.006	0.052	2.496	2.281	0.	1.854

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	33	14	23	43	0	24
normalized size	1	1.	2.06	0.88	1.44	2.69	0.	1.5
time (sec)	N/A	0.004	0.013	0.052	1.191	2.478	0.	1.262

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	48	0	0	0	0	0
normalized size	1	1.	0.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.717	0.016	0.456	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	45	0	0	0	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.449	0.01	0.435	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	43	0	0	0	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.394	0.011	0.651	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	47	0	0	0	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.457	0.012	0.613	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	50	0	0	0	0	0
normalized size	1	1.	0.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.51	0.013	1.226	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	842	842	48	0	0	0	0	0
normalized size	1	1.	0.06	0.	0.	0.	0.	0.
time (sec)	N/A	1.058	0.014	0.574	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	781	781	45	0	0	0	0	0
normalized size	1	1.	0.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.947	0.009	0.61	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	715	715	45	0	0	0	0	0
normalized size	1	1.	0.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.86	0.01	0.699	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	773	773	45	0	0	0	0	0
normalized size	1	1.	0.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.947	0.012	0.928	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	838	838	50	0	0	0	0	0
normalized size	1	1.	0.06	0.	0.	0.	0.	0.
time (sec)	N/A	1.036	0.012	1.415	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	48	0	0	0	0	0
normalized size	1	1.	0.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.015	0.477	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	45	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.013	0.656	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	45	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.01	0.401	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	45	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.01	0.608	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	43	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.01	0.678	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	45	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.013	0.861	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	50	0	0	0	0	0
normalized size	1	1.	0.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.013	1.42	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	50	0	0	0	0	0
normalized size	1	1.	0.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.012	0.652	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	45	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.01	0.01	0.441	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	58	96	49	58
normalized size	1	1.	1.	0.86	1.14	1.88	0.96	1.14
time (sec)	N/A	0.017	0.002	0.045	1.168	1.797	0.074	1.232

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	42	66	32	42
normalized size	1	1.	1.	0.91	1.2	1.89	0.91	1.2
time (sec)	N/A	0.011	0.001	0.046	1.14	1.86	0.067	1.196

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	47	22	28
normalized size	1	1.	1.	0.88	1.12	1.88	0.88	1.12
time (sec)	N/A	0.007	0.001	0.051	1.236	1.833	0.084	1.315

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	23	8	14
normalized size	1	1.	1.	0.92	1.17	1.92	0.67	1.17
time (sec)	N/A	0.002	0.	0.046	1.154	1.813	0.071	1.227

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	151	53	20
normalized size	1	1.	1.	0.67	0.	6.29	2.21	0.83
time (sec)	N/A	0.005	0.005	0.047	0.	2.112	0.154	1.237

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	261	78	47
normalized size	1	1.	1.	0.8	0.	5.8	1.73	1.04
time (sec)	N/A	0.01	0.027	0.054	0.	2.164	0.393	1.291

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	55	51	0	401	105	61
normalized size	1	1.	0.89	0.82	0.	6.47	1.69	0.98
time (sec)	N/A	0.016	0.038	0.05	0.	2.076	0.503	1.196

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	76	66	0	347	97	85
normalized size	1	1.	0.9	0.79	0.	4.13	1.15	1.01
time (sec)	N/A	0.022	0.111	0.047	0.	2.337	5.604	1.211

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	51	0	294	70	66
normalized size	1	1.	1.	0.78	0.	4.52	1.08	1.02
time (sec)	N/A	0.014	0.089	0.051	0.	2.363	3.863	1.228

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	49	36	0	232	41	50
normalized size	1	1.	1.07	0.78	0.	5.04	0.89	1.09
time (sec)	N/A	0.009	0.021	0.047	0.	2.389	2.716	1.245

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	0	153	17	31
normalized size	1	1.	1.	0.84	0.	6.12	0.68	1.24
time (sec)	N/A	0.005	0.006	0.046	0.	2.138	1.592	1.343

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	47	17	19
normalized size	1	1.	1.	0.94	1.19	2.94	1.06	1.19
time (sec)	N/A	0.002	0.004	0.047	1.059	2.133	0.849	1.234

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	42	99	95	36
normalized size	1	1.	0.74	0.67	1.08	2.54	2.44	0.92
time (sec)	N/A	0.006	0.009	0.049	1.135	2.261	1.182	1.239

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	40	37	62	146	413	55
normalized size	1	1.	0.69	0.64	1.07	2.52	7.12	0.95
time (sec)	N/A	0.01	0.011	0.048	1.068	2.259	2.289	1.241

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	51	48	82	192	1265	74
normalized size	1	1.	0.66	0.62	1.06	2.49	16.43	0.96
time (sec)	N/A	0.016	0.014	0.05	1.137	2.306	4.732	1.197

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	35	41	46	0	69
normalized size	1	1.	0.87	1.52	1.78	2.	0.	3.
time (sec)	N/A	0.003	0.009	0.071	1.701	2.087	0.	1.204

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	25	41	20	0	35
normalized size	1	1.	1.09	1.09	1.78	0.87	0.	1.52
time (sec)	N/A	0.003	0.005	0.069	1.766	2.135	0.	1.244

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	8	24	0	34
normalized size	1	1.	0.9	0.79	0.28	0.83	0.	1.17
time (sec)	N/A	0.005	0.006	0.126	1.67	2.185	0.	1.227

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	20	17	12	34	0	0
normalized size	1	1.	0.8	0.68	0.48	1.36	0.	0.
time (sec)	N/A	0.003	0.006	0.076	1.699	2.056	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	25	41	20	8	35
normalized size	1	1.	1.09	1.09	1.78	0.87	0.35	1.52
time (sec)	N/A	0.003	0.011	0.075	1.699	2.03	0.09	1.258

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	8	24	7	20
normalized size	1	1.	0.9	0.79	0.28	0.83	0.24	0.69
time (sec)	N/A	0.005	0.011	0.125	1.673	2.066	0.095	1.625

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	27	27	41	26	0	35
normalized size	1	1.	1.17	1.17	1.78	1.13	0.	1.52
time (sec)	N/A	0.003	0.008	0.066	1.696	2.046	0.	1.331

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	28	25	8	28	0	31
normalized size	1	1.	0.97	0.86	0.28	0.97	0.	1.07
time (sec)	N/A	0.005	0.007	0.11	1.631	2.103	0.	1.201

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	27	27	41	26	0	35
normalized size	1	1.	1.17	1.17	1.78	1.13	0.	1.52
time (sec)	N/A	0.003	0.006	0.069	1.736	2.039	0.	1.238

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	28	25	8	28	0	31
normalized size	1	1.	0.97	0.86	0.28	0.97	0.	1.07
time (sec)	N/A	0.006	0.004	0.122	1.699	2.021	0.	1.358

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	207	636	316	585	253	451
normalized size	1	1.	1.9	5.83	2.9	5.37	2.32	4.14
time (sec)	N/A	0.14	0.034	0.079	1.125	2.103	0.21	1.336

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	207	636	316	632	250	451
normalized size	1	1.	1.9	5.83	2.9	5.8	2.29	4.14
time (sec)	N/A	0.137	0.045	0.073	1.077	2.138	0.267	1.287

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	199	636	316	618	253	451
normalized size	1	1.	1.83	5.83	2.9	5.67	2.32	4.14
time (sec)	N/A	0.142	0.031	0.072	1.143	2.208	0.292	1.372

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	207	636	316	676	248	451
normalized size	1	1.	1.9	5.83	2.9	6.2	2.28	4.14
time (sec)	N/A	0.14	0.044	0.069	1.153	2.105	0.271	1.259

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	58	22	22
normalized size	1	1.	1.	0.94	1.22	3.22	1.22	1.22
time (sec)	N/A	0.011	0.005	0.044	1.781	2.135	0.104	1.24

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	28	7	11
normalized size	1	1.	1.	0.75	0.92	2.33	0.58	0.92
time (sec)	N/A	0.009	0.014	0.052	1.701	2.276	0.248	1.306

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	34	17	36	109	39	42
normalized size	1	1.	1.79	0.89	1.89	5.74	2.05	2.21
time (sec)	N/A	0.019	0.02	0.046	1.498	2.199	0.146	1.233

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	36	10	20
normalized size	1	1.	1.	1.08	1.38	2.77	0.77	1.54
time (sec)	N/A	0.005	0.003	0.048	1.229	2.126	0.119	1.259

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	47	14	23
normalized size	1	1.	1.	0.76	0.95	2.24	0.67	1.1
time (sec)	N/A	0.006	0.003	0.049	1.093	2.141	0.163	1.208

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	17	17	14	18	46	12	20
normalized size	1	2.83	2.83	2.33	3.	7.67	2.	3.33
time (sec)	N/A	0.004	0.003	0.047	1.137	2.095	0.095	1.261

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	51	130	76	54
normalized size	1	1.	1.	0.89	1.89	4.81	2.81	2.
time (sec)	N/A	0.019	0.009	0.045	1.122	2.275	0.193	1.226

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	26	53	131	76	61
normalized size	1	1.	1.07	0.96	1.96	4.85	2.81	2.26
time (sec)	N/A	0.019	0.008	0.044	1.148	2.189	0.32	1.273

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	36	108	87	36
normalized size	1	1.	1.	0.9	1.16	3.48	2.81	1.16
time (sec)	N/A	0.02	0.01	0.045	1.109	2.312	0.306	1.245

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	26	53	135	76	61
normalized size	1	1.	1.07	0.96	1.96	5.	2.81	2.26
time (sec)	N/A	0.018	0.008	0.054	1.218	2.359	0.292	1.312

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	0	259	124	46
normalized size	1	1.	1.	0.92	0.	6.82	3.26	1.21
time (sec)	N/A	0.034	0.011	0.223	0.	2.092	0.291	1.184

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	35	0	261	100	41
normalized size	1	1.	0.97	1.	0.	7.46	2.86	1.17
time (sec)	N/A	0.027	0.009	0.142	0.	2.176	0.235	1.317

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	41	31	0	149	102	74
normalized size	1	1.	1.28	0.97	0.	4.66	3.19	2.31
time (sec)	N/A	0.025	0.01	0.136	0.	2.099	0.323	1.379

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	37	49	126	39	49
normalized size	1	1.	1.	0.86	1.14	2.93	0.91	1.14
time (sec)	N/A	0.015	0.023	0.048	1.71	2.042	0.195	1.279

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	62	37	63	181	58	69
normalized size	1	1.	1.44	0.86	1.47	4.21	1.35	1.6
time (sec)	N/A	0.016	0.029	0.049	2.847	2.111	0.163	1.297

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	32	46	132	29	49
normalized size	1	1.	0.97	0.94	1.35	3.88	0.85	1.44
time (sec)	N/A	0.008	0.011	0.056	1.906	2.152	0.267	1.273

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	46	142	32	49
normalized size	1	1.	1.	0.76	1.1	3.38	0.76	1.17
time (sec)	N/A	0.009	0.013	0.056	1.186	2.052	0.192	1.232

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	70	68	0	724	265	90
normalized size	1	1.	0.99	0.96	0.	10.2	3.73	1.27
time (sec)	N/A	0.04	0.065	0.191	0.	2.199	1.171	1.24

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	86	0	664	228	101
normalized size	1	1.	1.	1.19	0.	9.22	3.17	1.4
time (sec)	N/A	0.036	0.043	0.152	0.	2.414	1.393	1.197

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	78	84	0	360	218	122
normalized size	1	1.	1.13	1.22	0.	5.22	3.16	1.77
time (sec)	N/A	0.034	0.061	0.149	0.	2.504	1.036	1.216

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	65	111	215	161	212	135
normalized size	1	1.	1.05	1.79	3.47	2.6	3.42	2.18
time (sec)	N/A	0.157	0.095	0.423	1.706	2.676	1.401	1.367

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	58	34	31	88	128	56	76
normalized size	1	1.76	1.03	0.94	2.67	3.88	1.7	2.3
time (sec)	N/A	0.036	0.032	0.178	1.147	2.706	0.57	1.264

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	58	32	31	82	128	56	73
normalized size	1	1.87	1.03	1.	2.65	4.13	1.81	2.35
time (sec)	N/A	0.029	0.031	0.147	1.223	2.359	0.451	1.214

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	33	36	57	165	36
normalized size	1	1.	1.12	1.94	2.12	3.35	9.71	2.12
time (sec)	N/A	0.018	0.02	0.064	1.795	2.25	0.244	1.406

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	23	23	56	39	45	69	70	45
normalized size	1	1.	2.43	1.7	1.96	3.	3.04	1.96
time (sec)	N/A	0.028	0.04	0.091	1.71	2.466	0.732	1.211

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	39	29	51	109	0	54
normalized size	1	1.	1.03	0.76	1.34	2.87	0.	1.42
time (sec)	N/A	0.011	0.017	0.049	1.619	2.41	0.	1.209

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	51	134	0	32
normalized size	1	1.	1.	0.83	1.7	4.47	0.	1.07
time (sec)	N/A	0.01	0.013	0.048	1.701	1.908	0.	1.261

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	50	70	109	0	55
normalized size	1	1.	1.	1.02	1.43	2.22	0.	1.12
time (sec)	N/A	0.01	0.016	0.049	1.49	2.165	0.	1.215

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	46	35	62	158	0	72
normalized size	1	1.	1.02	0.78	1.38	3.51	0.	1.6
time (sec)	N/A	0.015	0.017	0.044	1.708	1.964	0.	1.213

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	46	35	62	166	0	49
normalized size	1	1.	1.02	0.78	1.38	3.69	0.	1.09
time (sec)	N/A	0.016	0.019	0.047	1.718	1.944	0.	1.266

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	55	50	78	167	0	73
normalized size	1	1.	0.89	0.81	1.26	2.69	0.	1.18
time (sec)	N/A	0.015	0.028	0.045	1.74	1.975	0.	1.251

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	44	32	55	170	0	42
normalized size	1	1.	1.02	0.74	1.28	3.95	0.	0.98
time (sec)	N/A	0.012	0.019	0.046	1.719	1.901	0.	1.298

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	50	78	158	0	73
normalized size	1	1.	0.9	0.85	1.32	2.68	0.	1.24
time (sec)	N/A	0.013	0.02	0.052	1.69	1.856	0.	1.165

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	46	62	244	0	49
normalized size	1	1.	0.92	0.78	1.05	4.14	0.	0.83
time (sec)	N/A	0.016	0.025	0.047	1.724	2.009	0.	1.201

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	55	50	78	167	0	73
normalized size	1	1.	0.89	0.81	1.26	2.69	0.	1.18
time (sec)	N/A	0.013	0.027	0.043	1.668	2.046	0.	1.208

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	32	55	169	0	42
normalized size	1	1.	1.03	0.82	1.41	4.33	0.	1.08
time (sec)	N/A	0.008	0.019	0.045	1.743	1.966	0.	1.223

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	9	11	59	0	27
normalized size	1	1.	1.	0.64	0.79	4.21	0.	1.93
time (sec)	N/A	0.007	0.005	0.045	1.7	2.119	0.	1.27

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	14	7	11	88	0	8
normalized size	1	1.	1.4	0.7	1.1	8.8	0.	0.8
time (sec)	N/A	0.007	0.006	0.046	1.697	1.822	0.	1.16

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	30	30	59	0	28
normalized size	1	1.	0.96	1.2	1.2	2.36	0.	1.12
time (sec)	N/A	0.006	0.005	0.046	1.708	1.98	0.	1.196

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	22	105	0	45
normalized size	1	1.	1.	0.83	1.22	5.83	0.	2.5
time (sec)	N/A	0.01	0.006	0.048	1.765	1.912	0.	1.3

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	22	115	0	22
normalized size	1	1.	1.	0.79	1.16	6.05	0.	1.16
time (sec)	N/A	0.01	0.006	0.051	1.69	2.018	0.	1.267

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	30	38	111	0	46
normalized size	1	1.	0.8	0.86	1.09	3.17	0.	1.31
time (sec)	N/A	0.008	0.006	0.047	1.575	2.073	0.	1.288

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	15	115	0	15
normalized size	1	1.	1.	0.71	0.88	6.76	0.	0.88
time (sec)	N/A	0.007	0.006	0.049	1.705	1.946	0.	1.319

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	26	30	38	104	0	46
normalized size	1	1.	0.81	0.94	1.19	3.25	0.	1.44
time (sec)	N/A	0.008	0.007	0.049	1.741	1.769	0.	1.262

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	28	26	22	190	0	22
normalized size	1	1.	0.85	0.79	0.67	5.76	0.	0.67
time (sec)	N/A	0.008	0.007	0.057	1.824	1.921	0.	1.242

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	30	38	109	0	46
normalized size	1	1.	0.8	0.86	1.09	3.11	0.	1.31
time (sec)	N/A	0.008	0.006	0.051	1.841	1.989	0.	1.273

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	115	0	15
normalized size	1	1.	1.	0.92	1.15	8.85	0.	1.15
time (sec)	N/A	0.004	0.006	0.057	1.769	1.952	0.	1.3

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	51	0	320	0	66
normalized size	1	1.	1.	2.32	0.	14.55	0.	3.
time (sec)	N/A	0.013	0.027	0.264	0.	2.077	0.	1.438

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	44	0	323	0	72
normalized size	1	1.	1.	1.91	0.	14.04	0.	3.13
time (sec)	N/A	0.013	0.027	0.247	0.	1.98	0.	1.434

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	44	0	317	0	72
normalized size	1	1.	1.	2.2	0.	15.85	0.	3.6
time (sec)	N/A	0.011	0.026	0.242	0.	2.184	0.	1.278

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	24	35	95	0	23
normalized size	1	1.	1.	1.26	1.84	5.	0.	1.21
time (sec)	N/A	0.002	0.006	0.049	1.173	1.874	0.	1.193

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	28	41	112	0	23
normalized size	1	1.	1.	1.22	1.78	4.87	0.	1.
time (sec)	N/A	0.003	0.007	0.042	1.181	1.875	0.	1.281

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	41	70	0	39
normalized size	1	1.	1.	1.13	1.78	3.04	0.	1.7
time (sec)	N/A	0.005	0.03	0.045	1.182	1.929	0.	1.33

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	36	80	123	0	49
normalized size	1	1.	0.72	0.84	1.86	2.86	0.	1.14
time (sec)	N/A	0.007	0.009	0.046	1.142	2.062	0.	1.314

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	126	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.101	1.167	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.01	3.641	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.007	3.586	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.01	2.441	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.01	0.005	3.654	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.01	0.02	0.841	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	17	17	0	47	20	22
normalized size	1	1.	0.94	0.94	0.	2.61	1.11	1.22
time (sec)	N/A	0.002	0.007	0.044	0.	1.956	0.063	1.286

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	26	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.007	0.927	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	26	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.007	0.933	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.009	0.906	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.008	0.919	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.01	0.925	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [35] had the largest ratio of [0.5385]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	3	1.	13	0.231
2	A	6	3	1.	15	0.2
3	A	5	3	1.	15	0.2
4	A	4	3	1.	15	0.2
5	A	3	3	1.	15	0.2
6	A	6	3	1.	13	0.231
7	A	5	3	1.	13	0.231
8	A	4	3	1.	13	0.231
9	A	3	3	1.	13	0.231
10	A	3	3	1.	13	0.231
11	A	3	3	1.	13	0.231
12	A	4	3	1.	11	0.273
13	A	3	3	1.	11	0.273
14	A	3	3	1.	11	0.273
15	A	3	3	1.	11	0.273
16	A	3	2	1.	13	0.154
17	A	2	2	1.	15	0.133
18	A	1	1	1.	15	0.067
19	A	2	2	1.	15	0.133
20	A	3	2	1.	15	0.133
21	A	2	2	1.	13	0.154
22	A	1	1	1.	13	0.077
23	A	2	2	1.	13	0.154
24	A	3	2	1.	13	0.154
25	A	2	2	1.	16	0.125
26	A	2	2	1.	15	0.133
27	A	2	2	1.	13	0.154
28	A	2	2	1.	11	0.182
29	A	2	2	1.	11	0.182
30	A	6	5	1.	13	0.385
31	A	5	5	1.	13	0.385
32	A	4	4	1.	13	0.308
33	A	5	5	1.	13	0.385
34	A	6	5	1.	13	0.385
35	A	8	7	1.	13	0.538
36	A	7	7	1.	13	0.538
37	A	6	6	1.	13	0.462
38	A	7	7	1.	13	0.538
39	A	8	7	1.	13	0.538
40	A	5	4	1.	13	0.308
41	A	4	4	1.	13	0.308
42	A	4	4	1.	13	0.308
43	A	3	3	1.	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	3	3	1.	13	0.231
45	A	4	4	1.	13	0.308
46	A	5	4	1.	13	0.308
47	A	6	4	1.	13	0.308
48	A	1	1	1.	11	0.091
49	A	2	1	1.	9	0.111
50	A	2	1	1.	9	0.111
51	A	2	1	1.	9	0.111
52	A	1	0	1.	7	0.
53	A	1	1	1.	9	0.111
54	A	2	2	1.	9	0.222
55	A	3	2	1.	9	0.222
56	A	5	3	1.	11	0.273
57	A	4	3	1.	11	0.273
58	A	3	3	1.	11	0.273
59	A	2	2	1.	11	0.182
60	A	1	1	1.	11	0.091
61	A	2	2	1.	11	0.182
62	A	3	2	1.	11	0.182
63	A	4	2	1.	11	0.182
64	A	1	1	1.	14	0.071
65	A	1	1	1.	14	0.071
66	A	2	2	1.	14	0.143
67	A	1	1	1.	14	0.071
68	A	1	1	1.	14	0.071
69	A	2	2	1.	14	0.143
70	A	1	1	1.	14	0.071
71	A	2	2	1.	14	0.143
72	A	1	1	1.	14	0.071
73	A	2	2	1.	14	0.143
74	A	3	2	1.	23	0.087
75	A	3	2	1.	23	0.087
76	A	3	2	1.	23	0.087
77	A	3	2	1.	23	0.087
78	A	2	2	1.	12	0.167
79	A	2	2	1.	15	0.133
80	A	2	2	1.	12	0.167
81	A	3	2	1.	12	0.167
82	A	3	2	1.	12	0.167
83	B	3	2	2.83	10	0.2
84	A	2	2	1.	12	0.167
85	A	2	2	1.	12	0.167
86	A	2	2	1.	12	0.167
87	A	2	2	1.	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	2	2	1.	12	0.167
89	A	2	2	1.	13	0.154
90	A	2	2	1.	14	0.143
91	A	3	3	1.	12	0.25
92	A	3	3	1.	12	0.25
93	A	4	3	1.	12	0.25
94	A	4	3	1.	12	0.25
95	A	3	3	1.	12	0.25
96	A	3	3	1.	13	0.231
97	A	3	3	1.	14	0.214
98	A	2	2	1.	40	0.05
99	A	3	2	1.76	30	0.067
100	A	3	2	1.87	31	0.065
101	A	2	2	1.	14	0.143
102	A	2	2	1.	16	0.125
103	A	3	3	1.	14	0.214
104	A	3	3	1.	14	0.214
105	A	3	3	1.	14	0.214
106	A	3	3	1.	14	0.214
107	A	3	3	1.	14	0.214
108	A	3	3	1.	14	0.214
109	A	3	3	1.	14	0.214
110	A	3	3	1.	14	0.214
111	A	3	3	1.	14	0.214
112	A	3	3	1.	14	0.214
113	A	3	3	1.	14	0.214
114	A	2	2	1.	14	0.143
115	A	2	2	1.	14	0.143
116	A	2	2	1.	14	0.143
117	A	2	2	1.	14	0.143
118	A	2	2	1.	14	0.143
119	A	2	2	1.	14	0.143
120	A	2	2	1.	14	0.143
121	A	2	2	1.	14	0.143
122	A	2	2	1.	14	0.143
123	A	2	2	1.	14	0.143
124	A	2	2	1.	14	0.143
125	A	2	2	1.	27	0.074
126	A	2	2	1.	30	0.067
127	A	2	2	1.	28	0.071
128	A	1	1	1.	12	0.083
129	A	1	1	1.	14	0.071
130	A	1	1	1.	16	0.062
131	A	2	2	1.	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
132	A	1	1	1.	12	0.083
133	A	2	2	1.	12	0.167
134	A	2	2	1.	12	0.167
135	A	2	2	1.	12	0.167
136	A	2	2	1.	12	0.167
137	A	1	1	1.	10	0.1
138	A	1	1	1.	7	0.143
139	A	2	2	1.	12	0.167
140	A	2	2	1.	12	0.167
141	A	2	2	1.	12	0.167
142	A	2	2	1.	12	0.167
143	A	2	2	1.	12	0.167

Chapter 3

Listing of integrals

3.1 $\int \left(bx+cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=147

$$-\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4}+\frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3}-\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2}+\frac{35b^8\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{16384c^{9/2}}+\frac{(b+$$

```
[Out] (-35*b^6*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(16384*c^4) + (35*b^4*(b + 2*c*x)*(
b*x + c*x^2)^(3/2))/(6144*c^3) - (7*b^2*(b + 2*c*x)*(b*x + c*x^2)^(5/2))/(3
84*c^2) + ((b + 2*c*x)*(b*x + c*x^2)^(7/2))/(16*c) + (35*b^8*ArcTanh[(Sqrt[
c]*x)/Sqrt[b*x + c*x^2]])/(16384*c^(9/2))
```

Rubi [A] time = 0.0543799, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 620, 206}

$$-\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4}+\frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3}-\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2}+\frac{35b^8\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{16384c^{9/2}}+\frac{(b+$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(7/2),x]
```


```
[Out] (-35*b^6*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(16384*c^4) + (35*b^4*(b + 2*c*x)*(
b*x + c*x^2)^(3/2))/(6144*c^3) - (7*b^2*(b + 2*c*x)*(b*x + c*x^2)^(5/2))/(3
84*c^2) + ((b + 2*c*x)*(b*x + c*x^2)^(7/2))/(16*c) + (35*b^8*ArcTanh[(Sqrt[
c]*x)/Sqrt[b*x + c*x^2]])/(16384*c^(9/2))
```


Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(bx+cx^2\right)^{7/2} dx = \frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c} - \frac{\left(7b^2\right)\int \left(bx+cx^2\right)^{5/2} dx}{32c}$$
$$= -\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c} + \frac{\left(35b^4\right)\int \left(bx+cx^2\right)^{3/2} dx}{768c^2}$$
$$= \frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c} - \frac{\left(35b^6\right)\int \sqrt{bx}}{4096c}$$
$$= -\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4} + \frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2} + \frac{(b+2cx)(}{1}$$
$$= -\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4} + \frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2} + \frac{(b+2cx)(}{1}$$
$$= -\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4} + \frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2} + \frac{(b+2cx)(}{1}$$

Mathematica [A] time = 0.198729, size = 142, normalized size = 0.97

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(-56b^5c^2x^2+48b^4c^3x^3+10880b^3c^4x^4+25856b^2c^5x^5+70b^6cx-105b^7+21504bc^6x^6+6144c^7x^7\right)+\frac{105}{}\right.}{49152c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(-105*b^7 + 70*b^6*c*x - 56*b^5*c^2*x^2 + 48*b^
4*c^3*x^3 + 10880*b^3*c^4*x^4 + 25856*b^2*c^5*x^5 + 21504*b*c^6*x^6 + 6144*
c^7*x^7) + (105*b^(15/2)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[
1 + (c*x)/b])))/(49152*c^(9/2))
```

Maple [A] time = 0.05, size = 173, normalized size = 1.2

$$\frac{2\,cx+b}{16\,c}\left(cx^2+bx\right)^{\frac{7}{2}} - \frac{7\,b^2x}{192\,c}\left(cx^2+bx\right)^{\frac{5}{2}} - \frac{7\,b^3}{384\,c^2}\left(cx^2+bx\right)^{\frac{5}{2}} + \frac{35\,b^4x}{3072\,c^2}\left(cx^2+bx\right)^{\frac{3}{2}} + \frac{35\,b^5}{6144\,c^3}\left(cx^2+bx\right)^{\frac{3}{2}} - \frac{35\,b^6x}{8192\,c^3}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(7/2),x)
```

```
[Out] 1/16*(2*c*x+b)*(c*x^2+b*x)^(7/2)/c-7/192*b^2/c*(c*x^2+b*x)^(5/2)*x-7/384*b^
3/c^2*(c*x^2+b*x)^(5/2)+35/3072*b^4/c^2*(c*x^2+b*x)^(3/2)*x+35/6144*b^5/c^3
*(c*x^2+b*x)^(3/2)-35/8192*b^6/c^3*(c*x^2+b*x)^(1/2)*x-35/16384*b^7/c^4*(c*
x^2+b*x)^(1/2)+35/32768*b^8/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2
```

```
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.28795, size = 626, normalized size = 4.26

$$\left[\frac{105\,b^{8}\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^{2}+bx}\sqrt{c}\right)+2\left(6144\,c^{8}x^{7}+21504\,bc^{7}x^{6}+25856\,b^{2}c^{6}x^{5}+10880\,b^{3}c^{5}x^{4}+48\,b^{4}c^{4}x^{3}-\right.}{98304\,c^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/98304*(105*b^8*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*
(6144*c^8*x^7 + 21504*b*c^7*x^6 + 25856*b^2*c^6*x^5 + 10880*b^3*c^5*x^4 + 4
8*b^4*c^4*x^3 - 56*b^5*c^3*x^2 + 70*b^6*c^2*x - 105*b^7*c)*sqrt(c*x^2 + b*x
))/c^5, -1/49152*(105*b^8*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x))
 - (6144*c^8*x^7 + 21504*b*c^7*x^6 + 25856*b^2*c^6*x^5 + 10880*b^3*c^5*x^4
+ 48*b^4*c^4*x^3 - 56*b^5*c^3*x^2 + 70*b^6*c^2*x - 105*b^7*c)*sqrt(c*x^2 +
b*x))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx+cx^{2}\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(7/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(7/2), x)
```

Giac [A] time = 1.25027, size = 178, normalized size = 1.21

$$-\frac{35\,b^{8}\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^{2}+bx}\right)\sqrt{c}-b\right|\right)}{32768\,c^{\frac{9}{2}}}-\frac{1}{49152}\left(\frac{105\,b^{7}}{c^{4}}-2\left(\frac{35\,b^{6}}{c^{3}}-4\left(\frac{7\,b^{5}}{c^{2}}-2\left(\frac{3\,b^{4}}{c}+8\left(85\,b^{3}+2\left(101\,b^{2}c+\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(7/2),x, algorithm="giac")
```

```
[Out] -35/32768*b^8*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(9
/2) - 1/49152*(105*b^7/c^4 - 2*(35*b^6/c^3 - 4*(7*b^5/c^2 - 2*(3*b^4/c + 8*
(85*b^3 + 2*(101*b^2*c + 12*(2*c^3*x + 7*b*c^2)*x)*x)*x)*x)*x)*x)*sqrt(c*x^
2 + b*x)
```

3.2 $\int \left(3ix+4x^2\right)^{7/2} dx$

Optimal. Leaf size=121

$$\frac{1}{64}(8x+3i)\left(4x^2+3ix\right)^{7/2}+\frac{21(8x+3i)\left(4x^2+3ix\right)^{5/2}}{2048}+\frac{945(8x+3i)\left(4x^2+3ix\right)^{3/2}}{131072}+\frac{25515(8x+3i)\sqrt{4x^2+3ix}}{4194304}+$$

```
[Out] (25515*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/4194304 + (945*(3*I + 8*x)*((3*I)
*x + 4*x^2)^(3/2))/131072 + (21*(3*I + 8*x)*((3*I)*x + 4*x^2)^(5/2))/2048 +
 ((3*I + 8*x)*((3*I)*x + 4*x^2)^(7/2))/64 + ((229635*I)/16777216)*ArcSin[1
- ((8*I)/3)*x]
```

Rubi [A] time = 0.0308269, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {612, 619, 215}

$$\frac{1}{64}(8x+3i)\left(4x^2+3ix\right)^{7/2}+\frac{21(8x+3i)\left(4x^2+3ix\right)^{5/2}}{2048}+\frac{945(8x+3i)\left(4x^2+3ix\right)^{3/2}}{131072}+\frac{25515(8x+3i)\sqrt{4x^2+3ix}}{4194304}+$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(7/2),x]
```

```
[Out] (25515*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/4194304 + (945*(3*I + 8*x)*((3*I)
*x + 4*x^2)^(3/2))/131072 + (21*(3*I + 8*x)*((3*I)*x + 4*x^2)^(5/2))/2048 +
 ((3*I + 8*x)*((3*I)*x + 4*x^2)^(7/2))/64 + ((229635*I)/16777216)*ArcSin[1
- ((8*I)/3)*x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(3ix+4x^2\right)^{7/2} dx = \frac{1}{64}(3i+8x)\left(3ix+4x^2\right)^{7/2} + \frac{63}{128}\int \left(3ix+4x^2\right)^{5/2} dx$$

$$= \frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048} + \frac{1}{64}(3i+8x)\left(3ix+4x^2\right)^{7/2} + \frac{945\int \left(3ix+4x^2\right)^{3/2} dx}{4096}$$

$$= \frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072} + \frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048} + \frac{1}{64}(3i+8x)\left(3ix+4x^2\right)^{7/2} + \frac{25515\int}{2}$$

$$= \frac{25515(3i+8x)\sqrt{3ix+4x^2}}{4194304} + \frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072} + \frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048} + \frac{1}{64}(3i+$$

$$= \frac{25515(3i+8x)\sqrt{3ix+4x^2}}{4194304} + \frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072} + \frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048} + \frac{1}{64}(3i+$$

$$= \frac{25515(3i+8x)\sqrt{3ix+4x^2}}{4194304} + \frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072} + \frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048} + \frac{1}{64}(3i+$$

Mathematica [A] time = 0.0894461, size = 119, normalized size = 0.98

$$\frac{\sqrt{x(4x+3i)}\left(2\sqrt{3-4ix}\sqrt{x}\left(33554432x^7+88080384ix^6-79429632x^5-25067520ix^4+82944x^3-72576ix^2-68040x\right.\right.}{8388608\sqrt{3-4ix}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(7/2),x]
```

```
[Out] (Sqrt[x*(3*I + 4*x)]*(2*Sqrt[3 - (4*I)*x]*Sqrt[x]*(76545*I - 68040*x - (725
76*I)*x^2 + 82944*x^3 - (25067520*I)*x^4 - 79429632*x^5 + (88080384*I)*x^6
+ 33554432*x^7) - 229635*(-1)^(1/4)*ArcSin[(1 + I)*Sqrt[2/3]*Sqrt[x]]))/(83
88608*Sqrt[3 - (4*I)*x]*Sqrt[x])
```

Maple [A] time = 0.104, size = 91, normalized size = 0.8

$$\frac{3\,i+8\,x}{64}\left(3\,ix+4\,x^2\right)^{\frac{7}{2}} + \frac{63\,i+168\,x}{2048}\left(3\,ix+4\,x^2\right)^{\frac{5}{2}} + \frac{2835\,i+7560\,x}{131072}\left(3\,ix+4\,x^2\right)^{\frac{3}{2}} + \frac{76545\,i+204120\,x}{4194304}\sqrt{3\,ix+4\,x^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*I*x+4*x^2)^(7/2),x)
```

```
[Out] 1/64*(3*I+8*x)*(3*I*x+4*x^2)^(7/2)+21/2048*(3*I+8*x)*(3*I*x+4*x^2)^(5/2)+94
5/131072*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+25515/4194304*(3*I+8*x)*(3*I*x+4*x^2
)^(1/2)+229635/16777216*arcsinh(8/3*x+I)
```

Maxima [A] time = 1.89731, size = 176, normalized size = 1.45

$$\frac{1}{8}\left(4\,x^2+3i\,x\right)^{\frac{7}{2}}x + \frac{3}{64}i\left(4\,x^2+3i\,x\right)^{\frac{7}{2}} + \frac{21}{256}\left(4\,x^2+3i\,x\right)^{\frac{5}{2}}x + \frac{63}{2048}i\left(4\,x^2+3i\,x\right)^{\frac{5}{2}} + \frac{945}{16384}\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}x + \frac{2835}{131072}i\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/8*(4*x^2 + 3*I*x)^(7/2)*x + 3/64*I*(4*x^2 + 3*I*x)^(7/2) + 21/256*(4*x^2
+ 3*I*x)^(5/2)*x + 63/2048*I*(4*x^2 + 3*I*x)^(5/2) + 945/16384*(4*x^2 + 3*I
*x)^(3/2)*x + 2835/131072*I*(4*x^2 + 3*I*x)^(3/2) + 25515/524288*sqrt(4*x^2
 + 3*I*x)*x + 76545/4194304*I*sqrt(4*x^2 + 3*I*x) + 229635/16777216*log(8*x
 + 4*sqrt(4*x^2 + 3*I*x) + 3*I)
```

Fricas [A] time = 2.28309, size = 320, normalized size = 2.64

$$\frac{1}{268435456}\left(2147483648\,x^7 + 5637144576i\,x^6 - 5083496448\,x^5 - 1604321280i\,x^4 + 5308416\,x^3 - 4644864i\,x^2 - 435\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/268435456*(2147483648*x^7 + 5637144576*I*x^6 - 5083496448*x^5 - 160432128
0*I*x^4 + 5308416*x^3 - 4644864*I*x^2 - 4354560*x + 4898880*I)*sqrt(4*x^2 +
 3*I*x) - 229635/16777216*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - 1165671
/268435456
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4x^2 + 3ix\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(7/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4\,x^2 + 3i\,x\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 3*I*x)^(7/2), x)
```

3.3 $\int \left(3ix+4x^2\right)^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}+\frac{15(8x+3i)\left(4x^2+3ix\right)^{3/2}}{1024}+\frac{405(8x+3i)\sqrt{4x^2+3ix}}{32768}+\frac{3645i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{131072}$$

```
[Out] (405*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/32768 + (15*(3*I + 8*x)*((3*I)*x +
4*x^2)^(3/2))/1024 + ((3*I + 8*x)*((3*I)*x + 4*x^2)^(5/2))/48 + ((3645*I)/1
31072)*ArcSin[1 - ((8*I)/3)*x]
```

Rubi [A] time = 0.02172, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {612, 619, 215}

$$\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}+\frac{15(8x+3i)\left(4x^2+3ix\right)^{3/2}}{1024}+\frac{405(8x+3i)\sqrt{4x^2+3ix}}{32768}+\frac{3645i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{131072}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(5/2),x]
```

```
[Out] (405*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/32768 + (15*(3*I + 8*x)*((3*I)*x +
4*x^2)^(3/2))/1024 + ((3*I + 8*x)*((3*I)*x + 4*x^2)^(5/2))/48 + ((3645*I)/1
31072)*ArcSin[1 - ((8*I)/3)*x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(3ix+4x^2\right)^{5/2} dx = \frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2} + \frac{15}{32}\int \left(3ix+4x^2\right)^{3/2} dx$$
$$= \frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024} + \frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2} + \frac{405\int \sqrt{3ix+4x^2}\,dx}{2048}$$
$$= \frac{405(3i+8x)\sqrt{3ix+4x^2}}{32768} + \frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024} + \frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2} + \frac{3645\int \frac{}{\sqrt{}}}{655}$$
$$= \frac{405(3i+8x)\sqrt{3ix+4x^2}}{32768} + \frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024} + \frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2} + \frac{1215\,\mathrm{Sub}}{}$$
$$= \frac{405(3i+8x)\sqrt{3ix+4x^2}}{32768} + \frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024} + \frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2} + \frac{3645i\sin}{13}$$

Mathematica [A] time = 0.0870257, size = 88, normalized size = 0.93

$$\frac{\sqrt{x(4x+3i)}\left(524288x^5+983040ix^4-497664x^3-6912ix^2-6480x-\frac{10935\sqrt[4]{-1}\sin^{-1}\left((1+i)\sqrt{\frac{2}{3}}\sqrt{x}\right)}{\sqrt{3-4ix}\sqrt{x}}+7290i\right)}{196608}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(5/2),x]
```

```
[Out] (Sqrt[x*(3*I + 4*x)]*(7290*I - 6480*x - (6912*I)*x^2 - 497664*x^3 + (983040
*I)*x^4 + 524288*x^5 - (10935*(-1)^(1/4)*ArcSin[(1 + I)*Sqrt[2/3]*Sqrt[x]])
/(Sqrt[3 - (4*I)*x]*Sqrt[x])))/196608
```

Maple [A] time = 0.094, size = 71, normalized size = 0.8

$$\frac{3i+8x}{48}\left(3ix+4x^2\right)^{\frac{5}{2}} + \frac{45i+120x}{1024}\left(3ix+4x^2\right)^{\frac{3}{2}} + \frac{1215i+3240x}{32768}\sqrt{3ix+4x^2} + \frac{3645}{131072}\mathrm{Arcsinh}\left(\frac{8x}{3}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*I*x+4*x^2)^(5/2),x)
```

```
[Out] 1/48*(3*I+8*x)*(3*I*x+4*x^2)^(5/2)+15/1024*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+40
5/32768*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)+3645/131072*arcsinh(8/3*x+I)
```

Maxima [A] time = 1.78468, size = 139, normalized size = 1.46

$$\frac{1}{6}\left(4x^2+3ix\right)^{\frac{5}{2}}x + \frac{1}{16}i\left(4x^2+3ix\right)^{\frac{5}{2}} + \frac{15}{128}\left(4x^2+3ix\right)^{\frac{3}{2}}x + \frac{45}{1024}i\left(4x^2+3ix\right)^{\frac{3}{2}} + \frac{405}{4096}\sqrt{4x^2+3ix}x + \frac{1215}{32768}i\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(4*x^2 + 3*I*x)^(5/2)*x + 1/16*I*(4*x^2 + 3*I*x)^(5/2) + 15/128*(4*x^2
+ 3*I*x)^(3/2)*x + 45/1024*I*(4*x^2 + 3*I*x)^(3/2) + 405/4096*sqrt(4*x^2 +
3*I*x)*x + 1215/32768*I*sqrt(4*x^2 + 3*I*x) + 3645/131072*log(8*x + 4*sqrt(
4*x^2 + 3*I*x) + 3*I)
```

Fricas [A] time = 2.2787, size = 246, normalized size = 2.59

$$\frac{1}{3145728}\left(8388608\,x^5 + 15728640i\,x^4 - 7962624\,x^3 - 110592i\,x^2 - 103680\,x + 116640i\right)\sqrt{4\,x^2 + 3i\,x} - \frac{3645}{131072}\log\left(-2\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3145728*(8388608*x^5 + 15728640*I*x^4 - 7962624*x^3 - 110592*I*x^2 - 1036
80*x + 116640*I)*sqrt(4*x^2 + 3*I*x) - 3645/131072*log(-2*x + sqrt(4*x^2 +
3*I*x) - 3/4*I) - 8991/1048576
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4x^2 + 3ix\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4\,x^2 + 3i\,x\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 3*I*x)^(5/2), x)
```

3.4 $\int \left(3ix+4x^2\right)^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}+\frac{27(8x+3i)\sqrt{4x^2+3ix}}{1024}+\frac{243i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{4096}$$

```
[Out] (27*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/1024 + ((3*I + 8*x)*((3*I)*x + 4*x^2
)^(3/2))/32 + ((243*I)/4096)*ArcSin[1 - ((8*I)/3)*x]
```

Rubi [A] time = 0.0148143, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {612, 619, 215}

$$\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}+\frac{27(8x+3i)\sqrt{4x^2+3ix}}{1024}+\frac{243i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{4096}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(3/2),x]
```

```
[Out] (27*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/1024 + ((3*I + 8*x)*((3*I)*x + 4*x^2
)^(3/2))/32 + ((243*I)/4096)*ArcSin[1 - ((8*I)/3)*x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(3ix+4x^2\right)^{3/2}dx = \frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{27}{64}\int \sqrt{3ix+4x^2}\,dx$$
$$= \frac{27(3i+8x)\sqrt{3ix+4x^2}}{1024}+\frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{243\int \frac{1}{\sqrt{3ix+4x^2}}dx}{2048}$$
$$= \frac{27(3i+8x)\sqrt{3ix+4x^2}}{1024}+\frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{81\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{9}}}dx,x,3i+8x\right)}{4096}$$
$$= \frac{27(3i+8x)\sqrt{3ix+4x^2}}{1024}+\frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{243i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{4096}$$

Mathematica [A] time = 0.0703425, size = 76, normalized size = 1.1

$$\frac{\sqrt{x(4x+3i)}\left(2048x^3+2304ix^2-144x-\frac{243\sqrt[4]{-1}\sin^{-1}\left((1+i)\sqrt{\frac{2}{3}}\sqrt{x}\right)}{\sqrt{3-4ix}\sqrt{x}}+162i\right)}{2048}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(3/2),x]
```

```
[Out] (Sqrt[x*(3*I + 4*x)]*(162*I - 144*x + (2304*I)*x^2 + 2048*x^3 - (243*(-1)^(
1/4)*ArcSin[(1 + I)*Sqrt[2/3]*Sqrt[x]])/(Sqrt[3 - (4*I)*x]*Sqrt[x])))/2048
```

Maple [A] time = 0.095, size = 51, normalized size = 0.7

$$\frac{3\,i+8\,x}{32}\left(3\,ix+4\,x^2\right)^{\frac{3}{2}}+\frac{81\,i+216\,x}{1024}\sqrt{3\,ix+4\,x^2}+\frac{243}{4096}\text{Arcsinh}\left(\frac{8\,x}{3}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*I*x+4*x^2)^(3/2),x)
```

```
[Out] 1/32*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+27/1024*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)+24
3/4096*arcsinh(8/3*x+I)
```

Maxima [A] time = 1.71097, size = 103, normalized size = 1.49

$$\frac{1}{4}\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}x+\frac{3}{32}i\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}+\frac{27}{128}\sqrt{4\,x^2+3i\,x}x+\frac{81}{1024}i\sqrt{4\,x^2+3i\,x}+\frac{243}{4096}\log\left(8\,x+4\sqrt{4\,x^2+3i\,x}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*x^2 + 3*I*x)^(3/2)*x + 3/32*I*(4*x^2 + 3*I*x)^(3/2) + 27/128*sqrt(4*
x^2 + 3*I*x)*x + 81/1024*I*sqrt(4*x^2 + 3*I*x) + 243/4096*log(8*x + 4*sqrt(
4*x^2 + 3*I*x) + 3*I)
```

Fricas [A] time = 2.28958, size = 184, normalized size = 2.67

$$\frac{1}{32768}\left(32768\,x^3+36864i\,x^2-2304\,x+2592i\right)\sqrt{4\,x^2+3i\,x}-\frac{243}{4096}\,\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)-\frac{567}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32768*(32768*x^3 + 36864*I*x^2 - 2304*x + 2592*I)*sqrt(4*x^2 + 3*I*x) - 2
43/4096*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - 567/32768
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(4x^2+3ix\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 3*I*x)^(3/2), x)
```

3.5 $\int \sqrt{3ix+4x^2}\,dx$

Optimal. Leaf size=43

$$\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)+\frac{9}{64}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

```
[Out] ((3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/16 + ((9*I)/64)*ArcSin[1 - ((8*I)/3)*x]
```

Rubi [A] time = 0.0101643, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {612, 619, 215}

$$\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)+\frac{9}{64}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] ((3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/16 + ((9*I)/64)*ArcSin[1 - ((8*I)/3)*x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3ix+4x^2}\,dx &= \frac{1}{16}(3i+8x)\sqrt{3ix+4x^2}+\frac{9}{32}\int \frac{1}{\sqrt{3ix+4x^2}}\,dx\\ &= \frac{1}{16}(3i+8x)\sqrt{3ix+4x^2}+\frac{3}{64}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{9}}}\,dx,x,3i+8x\right)\\ &= \frac{1}{16}(3i+8x)\sqrt{3ix+4x^2}+\frac{9}{64}i\sin^{-1}\left(1-\frac{8ix}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0520036, size = 64, normalized size = 1.49

$$\frac{1}{32}\sqrt{x(4x+3i)}\left(16x-\frac{9\sqrt[4]{-1}\sin^{-1}\left((1+i)\sqrt{\frac{2}{3}}\sqrt{x}\right)}{\sqrt{3-4ix}\sqrt{x}}+6i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] (Sqrt[x*(3*I + 4*x)]*(6*I + 16*x - (9*(-1)^(1/4)*ArcSin[(1 + I)*Sqrt[2/3]*S
qrt[x]])/(Sqrt[3 - (4*I)*x]*Sqrt[x])))/32
```

Maple [A] time = 0.094, size = 31, normalized size = 0.7

$$\frac{3i+8x}{16}\sqrt{3ix+4x^2}+\frac{9}{64}\text{Arcsinh}\left(\frac{8x}{3}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*I*x+4*x^2)^(1/2),x)
```

```
[Out] 1/16*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)+9/64*arcsinh(8/3*x+I)
```

Maxima [A] time = 1.79029, size = 66, normalized size = 1.53

$$\frac{1}{2}\sqrt{4x^2+3ix}x+\frac{3}{16}i\sqrt{4x^2+3ix}+\frac{9}{64}\log\left(8x+4\sqrt{4x^2+3ix}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(4*x^2 + 3*I*x)*x + 3/16*I*sqrt(4*x^2 + 3*I*x) + 9/64*log(8*x + 4*s
qrt(4*x^2 + 3*I*x) + 3*I)
```

Fricas [A] time = 2.11607, size = 131, normalized size = 3.05

$$\frac{1}{256}\sqrt{4x^2+3ix}(128x+48i)-\frac{9}{64}\log\left(-2x+\sqrt{4x^2+3ix}-\frac{3}{4}i\right)-\frac{9}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/256*sqrt(4*x^2 + 3*I*x)*(128*x + 48*I) - 9/64*log(-2*x + sqrt(4*x^2 + 3*I
*x) - 3/4*I) - 9/256
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{4x^2+3ix}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(4*x**2 + 3*I*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\,x^2 + 3i\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x^2 + 3*I*x), x)
```

3.6 $\int \left(3x-4x^2\right)^{7/2} dx$

Optimal. Leaf size=101

$$-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{22963}{}$$

```
[Out] (-25515*(3 - 8*x)*Sqrt[3*x - 4*x^2])/4194304 - (945*(3 - 8*x)*(3*x - 4*x^2)
^(3/2))/131072 - (21*(3 - 8*x)*(3*x - 4*x^2)^(5/2))/2048 - ((3 - 8*x)*(3*x
- 4*x^2)^(7/2))/64 - (229635*ArcSin[1 - (8*x)/3])/16777216
```

Rubi [A] time = 0.0275681, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{22963}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(7/2),x]
```

```
[Out] (-25515*(3 - 8*x)*Sqrt[3*x - 4*x^2])/4194304 - (945*(3 - 8*x)*(3*x - 4*x^2)
^(3/2))/131072 - (21*(3 - 8*x)*(3*x - 4*x^2)^(5/2))/2048 - ((3 - 8*x)*(3*x
- 4*x^2)^(7/2))/64 - (229635*ArcSin[1 - (8*x)/3])/16777216
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \left(3x-4x^2\right)^{7/2} dx = -\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2} + \frac{63}{128}\int \left(3x-4x^2\right)^{5/2} dx$$
$$= -\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048} - \frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2} + \frac{945\int \left(3x-4x^2\right)^{3/2} dx}{4096}$$
$$= -\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072} - \frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048} - \frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2} + \frac{25515\int \sqrt{3x\cdots}}{262144\cdots}$$
$$= -\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304} - \frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072} - \frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048} - \frac{1}{64}(3-8x)\left(3\cdots\right.$$

$$= -\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304} - \frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072} - \frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048} - \frac{1}{64}(3-8x)\left(3\cdots\right.$$
$$= -\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304} - \frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072} - \frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048} - \frac{1}{64}(3-8x)\left(3\cdots\right.$$

Mathematica [A] time = 0.0722242, size = 88, normalized size = 0.87

$$\frac{2x\left(134217728x^8-452984832x^7+581959680x^6-338558976x^5+75534336x^4+41472x^3+54432x^2+102060x-229\cdots\right.}{8388608\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(7/2),x]
```

```
[Out] (2*x*(-229635 + 102060*x + 54432*x^2 + 41472*x^3 + 75534336*x^4 - 338558976
*x^5 + 581959680*x^6 - 452984832*x^7 + 134217728*x^8) - 229635*Sqrt[3 - 4*x
]*Sqrt[x]*ArcSin[Sqrt[1 - (4*x)/3]])/(8388608*Sqrt[-(x*(-3 + 4*x))])
```

Maple [A] time = 0.045, size = 82, normalized size = 0.8

$$-\frac{2835-7560\,x}{131072}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}-\frac{63-168\,x}{2048}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}-\frac{3-8\,x}{64}\left(-4\,x^2+3\,x\right)^{\frac{7}{2}}+\frac{229635}{16777216}\arcsin\left(-1+\frac{8\,x}{3}\right)-\frac{76\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+3*x)^(7/2),x)
```

```
[Out] -945/131072*(3-8*x)*(-4*x^2+3*x)^(3/2)-21/2048*(3-8*x)*(-4*x^2+3*x)^(5/2)-1
/64*(3-8*x)*(-4*x^2+3*x)^(7/2)+229635/16777216*arcsin(-1+8/3*x)-25515/41943
04*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

Maxima [A] time = 1.72491, size = 158, normalized size = 1.56

$$\frac{1}{8}\left(-4\,x^2+3\,x\right)^{\frac{7}{2}}x-\frac{3}{64}\left(-4\,x^2+3\,x\right)^{\frac{7}{2}}+\frac{21}{256}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}x-\frac{63}{2048}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}+\frac{945}{16384}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}x-\frac{2835}{131072}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/8*(-4*x^2 + 3*x)^(7/2)*x - 3/64*(-4*x^2 + 3*x)^(7/2) + 21/256*(-4*x^2 + 3
*x)^(5/2)*x - 63/2048*(-4*x^2 + 3*x)^(5/2) + 945/16384*(-4*x^2 + 3*x)^(3/2)
*x - 2835/131072*(-4*x^2 + 3*x)^(3/2) + 25515/524288*sqrt(-4*x^2 + 3*x)*x -
 76545/4194304*sqrt(-4*x^2 + 3*x) - 229635/16777216*arcsin(-8/3*x + 1)
```

Fricas [A] time = 2.23987, size = 247, normalized size = 2.45

$$-\frac{1}{4194304}\left(33554432\,x^7 - 88080384\,x^6 + 79429632\,x^5 - 25067520\,x^4 + 82944\,x^3 + 72576\,x^2 + 68040\,x + 76545\right)\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/4194304*(33554432*x^7 - 88080384*x^6 + 79429632*x^5 - 25067520*x^4 + 829
44*x^3 + 72576*x^2 + 68040*x + 76545)*sqrt(-4*x^2 + 3*x) - 229635/8388608*a
rctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-4x^2 + 3x\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(7/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(7/2), x)
```

Giac [A] time = 1.30064, size = 77, normalized size = 0.76

$$-\frac{1}{4194304}\,(8\,(16\,(8\,(32\,(8\,(16\,(8\,x - 21)x + 303)x - 765)x + 81)x + 567)x + 8505)x + 76545)\sqrt{-4\,x^2 + 3\,x} + \frac{229635}{1677721}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4194304*(8*(16*(8*(32*(8*(16*(8*x - 21)*x + 303)*x - 765)*x + 81)*x + 56
7)*x + 8505)*x + 76545)*sqrt(-4*x^2 + 3*x) + 229635/16777216*arcsin(8/3*x -
 1)
```

3.7 $\int \left(3x-4x^2\right)^{5/2} \, dx$

Optimal. Leaf size=79

$$-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}-\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024}-\frac{405(3-8x)\sqrt{3x-4x^2}}{32768}-\frac{3645\sin^{-1}\left(1-\frac{8x}{3}\right)}{131072}$$

```
[Out] (-405*(3 - 8*x)*Sqrt[3*x - 4*x^2])/32768 - (15*(3 - 8*x)*(3*x - 4*x^2)^(3/2
))/1024 - ((3 - 8*x)*(3*x - 4*x^2)^(5/2))/48 - (3645*ArcSin[1 - (8*x)/3])/1
31072
```

Rubi [A] time = 0.0192173, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}-\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024}-\frac{405(3-8x)\sqrt{3x-4x^2}}{32768}-\frac{3645\sin^{-1}\left(1-\frac{8x}{3}\right)}{131072}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(5/2),x]
```

```
[Out] (-405*(3 - 8*x)*Sqrt[3*x - 4*x^2])/32768 - (15*(3 - 8*x)*(3*x - 4*x^2)^(3/2
))/1024 - ((3 - 8*x)*(3*x - 4*x^2)^(5/2))/48 - (3645*ArcSin[1 - (8*x)/3])/1
31072
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \left(3x-4x^2\right)^{5/2} dx = -\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2} + \frac{15}{32}\int \left(3x-4x^2\right)^{3/2} dx$$
$$= -\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024} - \frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2} + \frac{405\int \sqrt{3x-4x^2}\,dx}{2048}$$
$$= -\frac{405(3-8x)\sqrt{3x-4x^2}}{32768} - \frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024} - \frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2} + \frac{3645\int \frac{1}{\sqrt{3x-4x}}}{65536}$$
$$= -\frac{405(3-8x)\sqrt{3x-4x^2}}{32768} - \frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024} - \frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2} - \frac{1215\,\mathrm{Subst}\left(\int\right.}{}$$
$$= -\frac{405(3-8x)\sqrt{3x-4x^2}}{32768} - \frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024} - \frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2} - \frac{3645\sin^{-1}\left(1\right.}{131072}$$

Mathematica [A] time = 0.0526547, size = 78, normalized size = 0.99

$$\frac{2x\left(-1048576x^6+2752512x^5-2469888x^4+760320x^3+2592x^2+4860x-10935\right)-10935\sqrt{3-4x}\sqrt{x}\sin^{-1}\left(\sqrt{1-\right.}{196608\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(5/2),x]
```

```
[Out] (2*x*(-10935 + 4860*x + 2592*x^2 + 760320*x^3 - 2469888*x^4 + 2752512*x^5 -
 1048576*x^6) - 10935*Sqrt[3 - 4*x]*Sqrt[x]*ArcSin[Sqrt[1 - (4*x)/3]])/(196
608*Sqrt[-(x*(-3 + 4*x))])
```

Maple [A] time = 0.044, size = 64, normalized size = 0.8

$$-\frac{45-120\,x}{1024}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}} - \frac{3-8\,x}{48}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}} + \frac{3645}{131072}\arcsin\left(-1+\frac{8\,x}{3}\right) - \frac{1215-3240\,x}{32768}\sqrt{-4\,x^2+3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+3*x)^(5/2),x)
```

```
[Out] -15/1024*(3-8*x)*(-4*x^2+3*x)^(3/2)-1/48*(3-8*x)*(-4*x^2+3*x)^(5/2)+3645/13
1072*arcsin(-1+8/3*x)-405/32768*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

Maxima [A] time = 1.71395, size = 122, normalized size = 1.54

$$\frac{1}{6}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}x - \frac{1}{16}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}} + \frac{15}{128}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}x - \frac{45}{1024}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}} + \frac{405}{4096}\sqrt{-4\,x^2+3\,x}\,x - \frac{1215}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-4*x^2 + 3*x)^(5/2)*x - 1/16*(-4*x^2 + 3*x)^(5/2) + 15/128*(-4*x^2 + 3
*x)^(3/2)*x - 45/1024*(-4*x^2 + 3*x)^(3/2) + 405/4096*sqrt(-4*x^2 + 3*x)*x
```

```
- 1215/32768*sqrt(-4*x^2 + 3*x) - 3645/131072*arcsin(-8/3*x + 1)
```

Fricas [A] time = 2.07362, size = 189, normalized size = 2.39

$$\frac{1}{98304}\left(262144\,x^{5}-491520\,x^{4}+248832\,x^{3}-3456\,x^{2}-3240\,x-3645\right)\sqrt{-4\,x^{2}+3\,x}-\frac{3645}{65536}\,\arctan\left(\frac{\sqrt{-4\,x^{2}+3\,x}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/98304*(262144*x^5 - 491520*x^4 + 248832*x^3 - 3456*x^2 - 3240*x - 3645)*s
qrt(-4*x^2 + 3*x) - 3645/65536*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-4x^{2}+3x\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(5/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(5/2), x)
```

Giac [A] time = 1.29414, size = 63, normalized size = 0.8

$$\frac{1}{98304}\,(8\,(16\,(8\,(32\,(8\,x-15)x+243)x-27)x-405)x-3645)\sqrt{-4\,x^{2}+3\,x}+\frac{3645}{131072}\,\arcsin\left(\frac{8}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/98304*(8*(16*(8*(32*(8*x - 15)*x + 243)*x - 27)*x - 405)*x - 3645)*sqrt(-
4*x^2 + 3*x) + 3645/131072*arcsin(8/3*x - 1)
```

3.8 $\int \left(3x-4x^2\right)^{3/2}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}-\frac{27(3-8x)\sqrt{3x-4x^2}}{1024}-\frac{243\sin^{-1}\left(1-\frac{8x}{3}\right)}{4096}$$

```
[Out] (-27*(3 - 8*x)*Sqrt[3*x - 4*x^2])/1024 - ((3 - 8*x)*(3*x - 4*x^2)^(3/2))/32
 - (243*ArcSin[1 - (8*x)/3])/4096
```

Rubi [A] time = 0.0128725, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}-\frac{27(3-8x)\sqrt{3x-4x^2}}{1024}-\frac{243\sin^{-1}\left(1-\frac{8x}{3}\right)}{4096}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(3/2),x]
```

```
[Out] (-27*(3 - 8*x)*Sqrt[3*x - 4*x^2])/1024 - ((3 - 8*x)*(3*x - 4*x^2)^(3/2))/32
 - (243*ArcSin[1 - (8*x)/3])/4096
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \left(3x-4x^2\right)^{3/2}\,dx = -\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2} + \frac{27}{64}\int \sqrt{3x-4x^2}\,dx$$
$$= -\frac{27(3-8x)\sqrt{3x-4x^2}}{1024} - \frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2} + \frac{243\int \frac{1}{\sqrt{3x-4x^2}}\,dx}{2048}$$
$$= -\frac{27(3-8x)\sqrt{3x-4x^2}}{1024} - \frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2} - \frac{81\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, 3-8x\right)}{4096}$$
$$= -\frac{27(3-8x)\sqrt{3x-4x^2}}{1024} - \frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2} - \frac{243\sin^{-1}\left(1-\frac{8x}{3}\right)}{4096}$$

Mathematica [A] time = 0.0508758, size = 68, normalized size = 1.19

$$\frac{2x\left(4096x^4-7680x^3+3744x^2+108x-243\right)-243\sqrt{3-4x}\sqrt{x}\sin^{-1}\left(\sqrt{1-\frac{4x}{3}}\right)}{2048\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(3/2),x]
```

```
[Out] (2*x*(-243 + 108*x + 3744*x^2 - 7680*x^3 + 4096*x^4) - 243*Sqrt[3 - 4*x]*Sq
rt[x]*ArcSin[Sqrt[1 - (4*x)/3]])/(2048*Sqrt[-(x*(-3 + 4*x))])
```

Maple [A] time = 0.044, size = 46, normalized size = 0.8

$$-\frac{3-8\,x}{32}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}+\frac{243}{4096}\arcsin\left(-1+\frac{8\,x}{3}\right)-\frac{81-216\,x}{1024}\sqrt{-4\,x^2+3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+3*x)^(3/2),x)
```

```
[Out] -1/32*(3-8*x)*(-4*x^2+3*x)^(3/2)+243/4096*arcsin(-1+8/3*x)-27/1024*(3-8*x)*
(-4*x^2+3*x)^(1/2)
```

Maxima [A] time = 1.82384, size = 85, normalized size = 1.49

$$\frac{1}{4}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}x-\frac{3}{32}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}+\frac{27}{128}\sqrt{-4\,x^2+3\,x}x-\frac{81}{1024}\sqrt{-4\,x^2+3\,x}-\frac{243}{4096}\arcsin\left(-\frac{8}{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-4*x^2 + 3*x)^(3/2)*x - 3/32*(-4*x^2 + 3*x)^(3/2) + 27/128*sqrt(-4*x^2
 + 3*x)*x - 81/1024*sqrt(-4*x^2 + 3*x) - 243/4096*arcsin(-8/3*x + 1)
```

Fricas [A] time = 2.18747, size = 143, normalized size = 2.51

$$-\frac{1}{1024}\left(1024\,x^{3}-1152\,x^{2}+72\,x+81\right)\sqrt{-4\,x^{2}+3\,x}-\frac{243}{2048}\,\arctan\left(\frac{\sqrt{-4\,x^{2}+3\,x}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1024*(1024*x^3 - 1152*x^2 + 72*x + 81)*sqrt(-4*x^2 + 3*x) - 243/2048*arc
tan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-4x^{2}+3x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(3/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(3/2), x)
```

Giac [A] time = 1.35534, size = 50, normalized size = 0.88

$$-\frac{1}{1024}\left(8\left(16\left(8\,x-9\right)x+9\right)x+81\right)\sqrt{-4\,x^{2}+3\,x}+\frac{243}{4096}\,\arcsin\left(\frac{8}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/1024*(8*(16*(8*x - 9)*x + 9)*x + 81)*sqrt(-4*x^2 + 3*x) + 243/4096*arcsi
n(8/3*x - 1)
```

3.9 $\int \sqrt{3x-4x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)-\frac{9}{64}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

```
[Out] -((3 - 8*x)*Sqrt[3*x - 4*x^2])/16 - (9*ArcSin[1 - (8*x)/3])/64
```

Rubi [A] time = 0.0091721, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)-\frac{9}{64}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*x - 4*x^2],x]
```

```
[Out] -((3 - 8*x)*Sqrt[3*x - 4*x^2])/16 - (9*ArcSin[1 - (8*x)/3])/64
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3x-4x^2}\,dx &= -\frac{1}{16}(3-8x)\sqrt{3x-4x^2}+\frac{9}{32}\int \frac{1}{\sqrt{3x-4x^2}}\,dx\\ &= -\frac{1}{16}(3-8x)\sqrt{3x-4x^2}-\frac{3}{64}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,3-8x\right)\\ &= -\frac{1}{16}(3-8x)\sqrt{3x-4x^2}-\frac{9}{64}\sin^{-1}\left(1-\frac{8x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0351241, size = 58, normalized size = 1.66

$$\frac{-2x\left(32x^2-36x+9\right)-9\sqrt{3-4x}\sqrt{x}\sin^{-1}\left(\sqrt{1-\frac{4x}{3}}\right)}{32\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*x - 4*x^2],x]
```

```
[Out] (-2*x*(9 - 36*x + 32*x^2) - 9*Sqrt[3 - 4*x]*Sqrt[x]*ArcSin[Sqrt[1 - (4*x)/3
]])/(32*Sqrt[-(x*(-3 + 4*x))])
```

Maple [A] time = 0.045, size = 28, normalized size = 0.8

$$\frac{9}{64}\arcsin\left(-1+\frac{8x}{3}\right)-\frac{3-8x}{16}\sqrt{-4x^2+3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+3*x)^(1/2),x)
```

```
[Out] 9/64*arcsin(-1+8/3*x)-1/16*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

Maxima [A] time = 1.75766, size = 49, normalized size = 1.4

$$\frac{1}{2}\sqrt{-4x^2+3x}\,x-\frac{3}{16}\sqrt{-4x^2+3x}-\frac{9}{64}\arcsin\left(-\frac{8}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 + 3*x)*x - 3/16*sqrt(-4*x^2 + 3*x) - 9/64*arcsin(-8/3*x + 1
)
```

Fricas [A] time = 2.18149, size = 101, normalized size = 2.89

$$\frac{1}{16}\sqrt{-4x^2+3x}(8x-3)-\frac{9}{32}\arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) - 9/32*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-4x^2+3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-4*x**2 + 3*x), x)
```

Giac [A] time = 1.30559, size = 36, normalized size = 1.03

$$\frac{1}{16}\sqrt{-4x^2+3x}(8x-3)+\frac{9}{64}\arcsin\left(\frac{8}{3}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) + 9/64*arcsin(8/3*x - 1)
```

3.10 $\int \sqrt{6x-x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{2}\sqrt{6x-x^2}(3-x)-\frac{9}{2}\sin^{-1}\left(1-\frac{x}{3}\right)$$

```
[Out] -((3 - x)*Sqrt[6*x - x^2])/2 - (9*ArcSin[1 - x/3])/2
```

Rubi [A] time = 0.0098087, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{2}\sqrt{6x-x^2}(3-x)-\frac{9}{2}\sin^{-1}\left(1-\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[6*x - x^2],x]

[Out] -((3 - x)*Sqrt[6*x - x^2])/2 - (9*ArcSin[1 - x/3])/2
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{6x-x^2}\,dx &= -\frac{1}{2}(3-x)\sqrt{6x-x^2}+\frac{9}{2}\int \frac{1}{\sqrt{6x-x^2}}\,dx\\ &= -\frac{1}{2}(3-x)\sqrt{6x-x^2}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx,x,6-2x\right)\\ &= -\frac{1}{2}(3-x)\sqrt{6x-x^2}-\frac{9}{2}\sin^{-1}\left(1-\frac{x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0427556, size = 32, normalized size = 0.91

$$\frac{1}{2}(x-3)\sqrt{-(x-6)x}-9\sin^{-1}\left(\sqrt{1-\frac{x}{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6*x - x^2],x]
```

```
[Out] ((-3 + x)*Sqrt[-((-6 + x)*x)])/2 - 9*ArcSin[Sqrt[1 - x/6]]
```

Maple [A] time = 0.056, size = 28, normalized size = 0.8

$$-\frac{-2x+6}{4}\sqrt{-x^2+6x}+\frac{9}{2}\arcsin\left(-1+\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+6*x)^(1/2),x)
```

```
[Out] -1/4*(-2*x+6)*(-x^2+6*x)^(1/2)+9/2*arcsin(-1+1/3*x)
```

Maxima [A] time = 1.78977, size = 49, normalized size = 1.4

$$\frac{1}{2}\sqrt{-x^2+6x}x-\frac{3}{2}\sqrt{-x^2+6x}-\frac{9}{2}\arcsin\left(-\frac{1}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 6*x)*x - 3/2*sqrt(-x^2 + 6*x) - 9/2*arcsin(-1/3*x + 1)
```

Fricas [A] time = 2.17013, size = 82, normalized size = 2.34

$$\frac{1}{2}\sqrt{-x^2+6x}(x-3)-9\arctan\left(\frac{\sqrt{-x^2+6x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 6*x)*(x - 3) - 9*arctan(sqrt(-x^2 + 6*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-x^2+6x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+6*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 + 6*x), x)
```

Giac [A] time = 1.28916, size = 34, normalized size = 0.97

$$\frac{1}{2}\sqrt{-x^2+6x}(x-3)+\frac{9}{2}\arcsin\left(\frac{1}{3}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+6*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 6*x)*(x - 3) + 9/2*arcsin(1/3*x - 1)
```

3.11 $\int \sqrt{5x-9x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{36}\sqrt{5x-9x^2}(5-18x)-\frac{25}{216}\sin^{-1}\left(1-\frac{18x}{5}\right)$$

```
[Out] -((5 - 18*x)*Sqrt[5*x - 9*x^2])/36 - (25*ArcSin[1 - (18*x)/5])/216
```

Rubi [A] time = 0.0097332, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{36}\sqrt{5x-9x^2}(5-18x)-\frac{25}{216}\sin^{-1}\left(1-\frac{18x}{5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5*x - 9*x^2],x]
```

```
[Out] -((5 - 18*x)*Sqrt[5*x - 9*x^2])/36 - (25*ArcSin[1 - (18*x)/5])/216
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{5x-9x^2}\,dx &= -\frac{1}{36}(5-18x)\sqrt{5x-9x^2}+\frac{25}{72}\int \frac{1}{\sqrt{5x-9x^2}}\,dx\\ &= -\frac{1}{36}(5-18x)\sqrt{5x-9x^2}-\frac{5}{216}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{25}}}\,dx,x,5-18x\right)\\ &= -\frac{1}{36}(5-18x)\sqrt{5x-9x^2}-\frac{25}{216}\sin^{-1}\left(1-\frac{18x}{5}\right)\end{aligned}$$

Mathematica [A] time = 0.0360542, size = 58, normalized size = 1.66

$$\frac{-3x\left(162x^2-135x+25\right)-25\sqrt{5-9x}\sqrt{x}\sin^{-1}\left(\sqrt{1-\frac{9x}{5}}\right)}{108\sqrt{-x(9x-5)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5*x - 9*x^2],x]
```

```
[Out] (-3*x*(25 - 135*x + 162*x^2) - 25*Sqrt[5 - 9*x]*Sqrt[x]*ArcSin[Sqrt[1 - (9*
x)/5]])/(108*Sqrt[-(x*(-5 + 9*x))])
```

Maple [A] time = 0.044, size = 28, normalized size = 0.8

$$\frac{25}{216}\arcsin\left(-1+\frac{18x}{5}\right)-\frac{5-18x}{36}\sqrt{-9x^2+5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-9*x^2+5*x)^(1/2),x)
```

```
[Out] 25/216*arcsin(-1+18/5*x)-1/36*(5-18*x)*(-9*x^2+5*x)^(1/2)
```

Maxima [A] time = 1.77616, size = 49, normalized size = 1.4

$$\frac{1}{2}\sqrt{-9x^2+5x}x-\frac{5}{36}\sqrt{-9x^2+5x}-\frac{25}{216}\arcsin\left(-\frac{18}{5}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x^2+5*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-9*x^2 + 5*x)*x - 5/36*sqrt(-9*x^2 + 5*x) - 25/216*arcsin(-18/5*x
+ 1)
```

Fricas [A] time = 2.07538, size = 105, normalized size = 3.

$$\frac{1}{36}\sqrt{-9x^2+5x}(18x-5)-\frac{25}{108}\arctan\left(\frac{\sqrt{-9x^2+5x}}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x^2+5*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(-9*x^2 + 5*x)*(18*x - 5) - 25/108*arctan(1/3*sqrt(-9*x^2 + 5*x)/x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-9x^2+5x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x**2+5*x)**(1/2),x)

[Out] Integral(sqrt(-9*x**2 + 5*x), x)
```

Giac [A] time = 1.22833, size = 36, normalized size = 1.03

$$\frac{1}{36}\sqrt{-9x^2+5x}(18x-5)+\frac{25}{216}\arcsin\left(\frac{18}{5}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x^2+5*x)^(1/2),x, algorithm="giac")

[Out] 1/36*sqrt(-9*x^2 + 5*x)*(18*x - 5) + 25/216*arcsin(18/5*x - 1)
```

3.12 $\int \left(x-x^2\right)^{3/2} \, dx$

Optimal. Leaf size=51

$$-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}-\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{3}{128}\sin^{-1}(1-2x)$$

```
[Out] (-3*(1 - 2*x)*Sqrt[x - x^2])/64 - ((1 - 2*x)*(x - x^2)^(3/2))/8 - (3*ArcSin
[1 - 2*x])/128
```

Rubi [A] time = 0.0101865, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {612, 619, 216}

$$-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}-\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{3}{128}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - x^2)^(3/2),x]
```

```
[Out] (-3*(1 - 2*x)*Sqrt[x - x^2])/64 - ((1 - 2*x)*(x - x^2)^(3/2))/8 - (3*ArcSin
[1 - 2*x])/128
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \left(x-x^2\right)^{3/2} \, dx &= -\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}+\frac{3}{16}\int \sqrt{x-x^2} \, dx\\
&= -\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}+\frac{3}{128}\int \frac{1}{\sqrt{x-x^2}} \, dx\\
&= -\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}-\frac{3}{128}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} \, dx,x,1-2x\right)\\
&= -\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}-\frac{3}{128}\sin^{-1}(1-2x)
\end{aligned}$$

Mathematica [A] time = 0.0669618, size = 44, normalized size = 0.86

$$\frac{1}{64}\left(-\sqrt{-(x-1)x}\left(16x^3-24x^2+2x+3\right)-3\sin^{-1}\left(\sqrt{1-x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - x^2)^(3/2),x]
```

```
[Out] (-(Sqrt[-((-1 + x)*x)]*(3 + 2*x - 24*x^2 + 16*x^3)) - 3*ArcSin[Sqrt[1 - x]]
)/64
```

Maple [A] time = 0.052, size = 42, normalized size = 0.8

$$-\frac{1-2\,x}{8}\left(-x^2+x\right)^{\frac{3}{2}}+\frac{3\arcsin\left(2\,x-1\right)}{128}-\frac{3-6\,x}{64}\sqrt{-x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+x)^(3/2),x)
```

```
[Out] -1/8*(1-2*x)*(-x^2+x)^(3/2)+3/128*arcsin(2*x-1)-3/64*(1-2*x)*(-x^2+x)^(1/2)
```

Maxima [A] time = 2.57555, size = 74, normalized size = 1.45

$$\frac{1}{4}\left(-x^2+x\right)^{\frac{3}{2}}x-\frac{1}{8}\left(-x^2+x\right)^{\frac{3}{2}}+\frac{3}{32}\sqrt{-x^2+x}x-\frac{3}{64}\sqrt{-x^2+x}+\frac{3}{128}\arcsin\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + x)^(3/2)*x - 1/8*(-x^2 + x)^(3/2) + 3/32*sqrt(-x^2 + x)*x - 3/6
4*sqrt(-x^2 + x) + 3/128*arcsin(2*x - 1)
```

Fricas [A] time = 2.09505, size = 111, normalized size = 2.18

$$-\frac{1}{64}\left(16\,x^3-24\,x^2+2\,x+3\right)\sqrt{-x^2+x}-\frac{3}{64}\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(16*x^3 - 24*x^2 + 2*x + 3)*sqrt(-x^2 + x) - 3/64*arctan(sqrt(-x^2 +
x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-x^2+x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+x)**(3/2),x)
```

```
[Out] Integral((-x**2 + x)**(3/2), x)
```

Giac [A] time = 1.37434, size = 47, normalized size = 0.92

$$-\frac{1}{64}\left(2\left(4\left(2x-3\right)x+1\right)x+3\right)\sqrt{-x^2+x}+\frac{3}{128}\arcsin\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/64*(2*(4*(2*x - 3)*x + 1)*x + 3)*sqrt(-x^2 + x) + 3/128*arcsin(2*x - 1)
```

3.13 $\int \sqrt{4x+x^2}\,dx$

Optimal. Leaf size=35

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-4\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

```
[Out] ((2 + x)*Sqrt[4*x + x^2])/2 - 4*ArcTanh[x/Sqrt[4*x + x^2]]
```

Rubi [A] time = 0.0071598, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {612, 620, 206}

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-4\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4*x + x^2],x]
```

```
[Out] ((2 + x)*Sqrt[4*x + x^2])/2 - 4*ArcTanh[x/Sqrt[4*x + x^2]]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{4x+x^2}\,dx &= \frac{1}{2}(2+x)\sqrt{4x+x^2}-2\int \frac{1}{\sqrt{4x+x^2}}\,dx\\ &= \frac{1}{2}(2+x)\sqrt{4x+x^2}-4\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{4x+x^2}}\right)\\ &= \frac{1}{2}(2+x)\sqrt{4x+x^2}-4\tanh^{-1}\left(\frac{x}{\sqrt{4x+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0298817, size = 40, normalized size = 1.14

$$\frac{1}{2}\sqrt{x(x+4)}\left(x-\frac{8\sinh^{-1}\left(\frac{\sqrt{x}}{2}\right)}{\sqrt{x+4}\sqrt{x}}+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4*x + x^2],x]
```

```
[Out] (Sqrt[x*(4 + x)]*(2 + x - (8*ArcSinh[Sqrt[x]/2])/(Sqrt[x]*Sqrt[4 + x])))/2
```

Maple [A] time = 0.052, size = 33, normalized size = 0.9

$$\frac{2x+4}{4}\sqrt{x^2+4x}-2\ln\left(x+2+\sqrt{x^2+4x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x)^(1/2),x)
```

```
[Out] 1/4*(2*x+4)*(x^2+4*x)^(1/2)-2*ln(x+2+(x^2+4*x)^(1/2))
```

Maxima [A] time = 1.12924, size = 55, normalized size = 1.57

$$\frac{1}{2}\sqrt{x^2+4x}x+\sqrt{x^2+4x}-2\log\left(2x+2\sqrt{x^2+4x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 4*x)*x + sqrt(x^2 + 4*x) - 2*log(2*x + 2*sqrt(x^2 + 4*x) + 4
)
```

Fricas [A] time = 2.16231, size = 85, normalized size = 2.43

$$\frac{1}{2}\sqrt{x^2+4x}(x+2)+2\log\left(-x+\sqrt{x^2+4x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(-x + sqrt(x^2 + 4*x) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^2+4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 4*x), x)
```

Giac [A] time = 1.25703, size = 45, normalized size = 1.29

$$\frac{1}{2}\sqrt{x^2+4x}(x+2)+2\log\left(\left|-x+\sqrt{x^2+4x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(abs(-x + sqrt(x^2 + 4*x) - 2))
```

3.14 $\int \sqrt{-8x+x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{2}\sqrt{x^2-8x}(4-x)-16\tanh^{-1}\left(\frac{x}{\sqrt{x^2-8x}}\right)$$

```
[Out] -((4 - x)*Sqrt[-8*x + x^2])/2 - 16*ArcTanh[x/Sqrt[-8*x + x^2]]
```

Rubi [A] time = 0.0066753, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {612, 620, 206}

$$-\frac{1}{2}\sqrt{x^2-8x}(4-x)-16\tanh^{-1}\left(\frac{x}{\sqrt{x^2-8x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-8*x + x^2],x]
```

```
[Out] -((4 - x)*Sqrt[-8*x + x^2])/2 - 16*ArcTanh[x/Sqrt[-8*x + x^2]]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{-8x+x^2}\,dx &= -\frac{1}{2}(4-x)\sqrt{-8x+x^2}-8\int \frac{1}{\sqrt{-8x+x^2}}\,dx\\ &= -\frac{1}{2}(4-x)\sqrt{-8x+x^2}-16\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-8x+x^2}}\right)\\ &= -\frac{1}{2}(4-x)\sqrt{-8x+x^2}-16\tanh^{-1}\left(\frac{x}{\sqrt{-8x+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0376307, size = 48, normalized size = 1.3

$$\frac{x\left(x^2-12x+32\right)+32\sqrt{-(x-8)x}\sin^{-1}\left(\sqrt{1-\frac{x}{8}}\right)}{2\sqrt{(x-8)x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-8*x + x^2],x]
```

```
[Out] (x*(32 - 12*x + x^2) + 32*Sqrt[-((-8 + x)*x)]*ArcSin[Sqrt[1 - x/8]])/(2*Sqr
t[(-8 + x)*x])
```

Maple [A] time = 0.051, size = 33, normalized size = 0.9

$$\frac{2x-8}{4}\sqrt{x^2-8x}-8\ln\left(x-4+\sqrt{x^2-8x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-8*x)^(1/2),x)
```

```
[Out] 1/4*(2*x-8)*(x^2-8*x)^(1/2)-8*ln(x-4+(x^2-8*x)^(1/2))
```

Maxima [A] time = 2.32576, size = 58, normalized size = 1.57

$$\frac{1}{2}\sqrt{x^2-8x}x-2\sqrt{x^2-8x}-8\log\left(2x+2\sqrt{x^2-8x}-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 - 8*x)*x - 2*sqrt(x^2 - 8*x) - 8*log(2*x + 2*sqrt(x^2 - 8*x) -
 8)
```

Fricas [A] time = 2.3514, size = 85, normalized size = 2.3

$$\frac{1}{2}\sqrt{x^2-8x}(x-4)+8\log\left(-x+\sqrt{x^2-8x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 - 8*x)*(x - 4) + 8*log(-x + sqrt(x^2 - 8*x) + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^2-8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-8*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 - 8*x), x)
```

Giac [A] time = 1.28506, size = 45, normalized size = 1.22

$$\frac{1}{2}\sqrt{x^2-8x}(x-4)+8\log\left(\left|-x+\sqrt{x^2-8x}+4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-8*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 - 8*x)*(x - 4) + 8*log(abs(-x + sqrt(x^2 - 8*x) + 4))
```

3.15 $\int \sqrt{-x+x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{4}\sqrt{x^2-x}(1-2x)-\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)$$

```
[Out] -((1 - 2*x)*Sqrt[-x + x^2])/4 - ArcTanh[x/Sqrt[-x + x^2]]/4
```

Rubi [A] time = 0.006844, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {612, 620, 206}

$$-\frac{1}{4}\sqrt{x^2-x}(1-2x)-\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x + x^2],x]
```

```
[Out] -((1 - 2*x)*Sqrt[-x + x^2])/4 - ArcTanh[x/Sqrt[-x + x^2]]/4
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{-x+x^2}\,dx &= -\frac{1}{4}(1-2x)\sqrt{-x+x^2}-\frac{1}{8}\int \frac{1}{\sqrt{-x+x^2}}\,dx\\ &= -\frac{1}{4}(1-2x)\sqrt{-x+x^2}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-x+x^2}}\right)\\ &= -\frac{1}{4}(1-2x)\sqrt{-x+x^2}-\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0291701, size = 46, normalized size = 1.18

$$\frac{2x^3-3x^2+x+\sqrt{-(x-1)x}\sin^{-1}\left(\sqrt{1-x}\right)}{4\sqrt{(x-1)x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-x + x^2],x]
```

```
[Out] (x - 3*x^2 + 2*x^3 + Sqrt[-((-1 + x)*x)]*ArcSin[Sqrt[1 - x]])/(4*Sqrt[(-1 +
 x)*x])
```

Maple [A] time = 0.049, size = 33, normalized size = 0.9

$$\frac{2x-1}{4}\sqrt{x^2-x}-\frac{1}{8}\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x)^(1/2),x)
```

```
[Out] 1/4*(2*x-1)*(x^2-x)^(1/2)-1/8*ln(x-1/2+(x^2-x)^(1/2))
```

Maxima [A] time = 1.22866, size = 58, normalized size = 1.49

$$\frac{1}{2}\sqrt{x^2-x}x-\frac{1}{4}\sqrt{x^2-x}-\frac{1}{8}\log\left(2x+2\sqrt{x^2-x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 - x)*x - 1/4*sqrt(x^2 - x) - 1/8*log(2*x + 2*sqrt(x^2 - x) - 1
)
```

Fricas [A] time = 2.17426, size = 90, normalized size = 2.31

$$\frac{1}{4}\sqrt{x^2-x}(2x-1)+\frac{1}{8}\log\left(-2x+2\sqrt{x^2-x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^2 - x)*(2*x - 1) + 1/8*log(-2*x + 2*sqrt(x^2 - x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^2-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 - x), x)
```

Giac [A] time = 1.32737, size = 50, normalized size = 1.28

$$\frac{1}{4}\sqrt{x^2-x}(2x-1)+\frac{1}{8}\log\left(\left|-2x+2\sqrt{x^2-x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^2 - x)*(2*x - 1) + 1/8*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))
```

3.16 $\int \frac{1}{(bx+cx^2)^{7/2}} \, dx$

Optimal. Leaf size=83

$$-\frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}}+\frac{32c(b+2cx)}{15b^4\left(bx+cx^2\right)^{3/2}}-\frac{2(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/2}}$$

```
[Out] (-2*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/2)) + (32*c*(b + 2*c*x))/(15*b^4*(
b*x + c*x^2)^(3/2)) - (256*c^2*(b + 2*c*x))/(15*b^6*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0184755, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {614, 613}

$$-\frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}}+\frac{32c(b+2cx)}{15b^4\left(bx+cx^2\right)^{3/2}}-\frac{2(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-7/2),x]
```

```
[Out] (-2*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/2)) + (32*c*(b + 2*c*x))/(15*b^4*(
b*x + c*x^2)^(3/2)) - (256*c^2*(b + 2*c*x))/(15*b^6*Sqrt[b*x + c*x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(bx+cx^2\right)^{7/2}} \, dx &= -\frac{2(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/2}}-\frac{(16c)\int \frac{1}{\left(bx+cx^2\right)^{5/2}} \, dx}{5b^2}\\
&= -\frac{2(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/2}}+\frac{32c(b+2cx)}{15b^4\left(bx+cx^2\right)^{3/2}}+\frac{\left(128c^2\right)\int \frac{1}{\left(bx+cx^2\right)^{3/2}} \, dx}{15b^4}\\
&= -\frac{2(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/2}}+\frac{32c(b+2cx)}{15b^4\left(bx+cx^2\right)^{3/2}}-\frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0248406, size = 70, normalized size = 0.84

$$-\frac{2\left(80b^3c^2x^2+480b^2c^3x^3-10b^4cx+3b^5+640bc^4x^4+256c^5x^5\right)}{15b^6(x(b+cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-7/2),x]
```

```
[Out] (-2*(3*b^5 - 10*b^4*c*x + 80*b^3*c^2*x^2 + 480*b^2*c^3*x^3 + 640*b*c^4*x^4
+ 256*c^5*x^5))/(15*b^6*(x*(b + c*x))^(5/2))
```

Maple [A] time = 0.051, size = 75, normalized size = 0.9

$$-\frac{2\,x\,(cx+b)\left(256\,c^5x^5+640\,c^4x^4b+480\,c^3x^3b^2+80\,c^2x^2b^3-10\,cxb^4+3\,b^5\right)}{15\,b^6}\left(cx^2+bx\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(7/2),x)
```

```
[Out] -2/15*x*(c*x+b)*(256*c^5*x^5+640*b*c^4*x^4+480*b^2*c^3*x^3+80*b^3*c^2*x^2-1
0*b^4*c*x+3*b^5)/b^6/(c*x^2+b*x)^(7/2)
```

Maxima [A] time = 1.30025, size = 150, normalized size = 1.81

$$-\frac{4\,cx}{5\left(cx^2+bx\right)^{\frac{5}{2}}b^2}+\frac{64\,c^2x}{15\left(cx^2+bx\right)^{\frac{3}{2}}b^4}-\frac{512\,c^3x}{15\,\sqrt{cx^2+bx}b^6}-\frac{2}{5\left(cx^2+bx\right)^{\frac{5}{2}}b}+\frac{32\,c}{15\left(cx^2+bx\right)^{\frac{3}{2}}b^3}-\frac{256\,c^2}{15\,\sqrt{cx^2+bx}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -4/5*c*x/((c*x^2 + b*x)^(5/2)*b^2) + 64/15*c^2*x/((c*x^2 + b*x)^(3/2)*b^4)
- 512/15*c^3*x/(sqrt(c*x^2 + b*x)*b^6) - 2/5/((c*x^2 + b*x)^(5/2)*b) + 32/1
5*c/((c*x^2 + b*x)^(3/2)*b^3) - 256/15*c^2/(sqrt(c*x^2 + b*x)*b^5)
```

Fricas [A] time = 2.22589, size = 223, normalized size = 2.69

$$-\frac{2\left(256\,c^5x^5+640\,bc^4x^4+480\,b^2c^3x^3+80\,b^3c^2x^2-10\,b^4cx+3\,b^5\right)\sqrt{cx^2+bx}}{15\left(b^6c^3x^6+3\,b^7c^2x^5+3\,b^8cx^4+b^9x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(256*c^5*x^5 + 640*b*c^4*x^4 + 480*b^2*c^3*x^3 + 80*b^3*c^2*x^2 - 10*
b^4*c*x + 3*b^5)*sqrt(c*x^2 + b*x)/(b^6*c^3*x^6 + 3*b^7*c^2*x^5 + 3*b^8*c*x
^4 + b^9*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx+cx^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(7/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(-7/2), x)
```

Giac [A] time = 1.30193, size = 100, normalized size = 1.2

$$-\frac{2\left(2\left(8\left(2\left(4x\left(\frac{2c^5x}{b^6}+\frac{5c^4}{b^5}\right)+\frac{15c^3}{b^4}\right)x+\frac{5c^2}{b^3}\right)x-\frac{5c}{b^2}\right)x+\frac{3}{b}\right)}{15\left(cx^2+bx\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(2*(8*(2*(4*x*(2*c^5*x/b^6 + 5*c^4/b^5) + 15*c^3/b^4)*x + 5*c^2/b^3)*
x - 5*c/b^2)*x + 3/b)/(c*x^2 + b*x)^(5/2)
```

3.17 $\int \frac{1}{\sqrt{3ix+4x^2}} \, dx$

Optimal. Leaf size=16

$$\frac{1}{2} i \sin^{-1}\left(1-\frac{8ix}{3}\right)$$

```
[Out] (I/2)*ArcSin[1 - ((8*I)/3)*x]
```

Rubi [A] time = 0.0064543, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {619, 215}

$$\frac{1}{2} i \sin^{-1}\left(1-\frac{8ix}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] (I/2)*ArcSin[1 - ((8*I)/3)*x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{3ix+4x^2}} \, dx &= \frac{1}{6} \text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{9}}} \, dx, x, 3i+8x\right) \\ &= \frac{1}{2} i \sin^{-1}\left(1-\frac{8ix}{3}\right) \end{aligned}$$

Mathematica [B] time = 0.0173975, size = 53, normalized size = 3.31

$$-\frac{(-1)^{3/4}\sqrt{3-4ix}\sqrt{x}\sin^{-1}\left((1+i)\sqrt{\frac{2}{3}}\sqrt{x}\right)}{\sqrt{x(4x+3i)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] -(((-1)^(3/4)*Sqrt[3 - (4*I)*x]*Sqrt[x]*ArcSin[(1 + I)*Sqrt[2/3]*Sqrt[x]])/
Sqrt[x*(3*I + 4*x)])
```

Maple [A] time = 0.105, size = 10, normalized size = 0.6

$$\frac{1}{2}\text{Arcsinh}\left(\frac{8\,x}{3}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*I*x+4*x^2)^(1/2),x)
```

```
[Out] 1/2*arcsinh(8/3*x+I)
```

Maxima [B] time = 1.79627, size = 28, normalized size = 1.75

$$\frac{1}{2}\,\log\left(8\,x+4\,\sqrt{4\,x^2+3i\,x}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)
```

Fricas [B] time = 2.4758, size = 62, normalized size = 3.88

$$-\frac{1}{2}\,\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{4x^2+3ix}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*x**2 + 3*I*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.18 $\int \frac{1}{\left(3ix+4x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\frac{2(8x+3i)}{9\sqrt{4x^2+3ix}}$$

```
[Out] (2*(3*I + 8*x))/(9*Sqrt[(3*I)*x + 4*x^2])
```

Rubi [A] time = 0.0025989, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {613}

$$\frac{2(8x+3i)}{9\sqrt{4x^2+3ix}}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(-3/2),x]

[Out] (2*(3*I + 8*x))/(9*Sqrt[(3*I)*x + 4*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(3ix+4x^2\right)^{3/2}}\,dx = \frac{2(3i+8x)}{9\sqrt{3ix+4x^2}}$$

Mathematica [A] time = 0.0053476, size = 24, normalized size = 0.92

$$\frac{2(8x+3i)}{9\sqrt{x(4x+3i)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(-3/2),x]

[Out] (2*(3*I + 8*x))/(9*Sqrt[x*(3*I + 4*x)])
```

Maple [A] time = 0.123, size = 21, normalized size = 0.8

$$\frac{6\,i+16\,x}{9}\frac{1}{\sqrt{3\,ix+4\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*I*x+4*x^2)^(3/2),x)

[Out] 2/9*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

Maxima [A] time = 1.222, size = 38, normalized size = 1.46

$$\frac{16\,x}{9\,\sqrt{4\,x^2+3i\,x}}+\frac{2i}{3\,\sqrt{4\,x^2+3i\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="maxima")

[Out] 16/9*x/sqrt(4*x^2 + 3*I*x) + 2/3*I/sqrt(4*x^2 + 3*I*x)
```

Fricas [B] time = 2.64556, size = 100, normalized size = 3.85

$$\frac{32\,x^2+\sqrt{4\,x^2+3i\,x}(16\,x+6i)+24i\,x}{9\left(4\,x^2+3i\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="fricas")

[Out] 1/9*(32*x^2 + sqrt(4*x^2 + 3*I*x)*(16*x + 6*I) + 24*I*x)/(4*x^2 + 3*I*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(4x^2+3ix\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x**2)**(3/2),x)

[Out] Integral((4*x**2 + 3*I*x)**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.19 $\int \frac{1}{\left(3ix+4x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=53

$$\frac{64(8x+3i)}{243\sqrt{4x^2+3ix}}+\frac{2(8x+3i)}{27\left(4x^2+3ix\right)^{3/2}}$$

```
[Out] (2*(3*I + 8*x))/(27*((3*I)*x + 4*x^2)^(3/2)) + (64*(3*I + 8*x))/(243*Sqrt[(
3*I)*x + 4*x^2])
```

Rubi [A] time = 0.0074662, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {614, 613}

$$\frac{64(8x+3i)}{243\sqrt{4x^2+3ix}}+\frac{2(8x+3i)}{27\left(4x^2+3ix\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(-5/2),x]
```

```
[Out] (2*(3*I + 8*x))/(27*((3*I)*x + 4*x^2)^(3/2)) + (64*(3*I + 8*x))/(243*Sqrt[(
3*I)*x + 4*x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(3ix+4x^2\right)^{5/2}}\,dx &= \frac{2(3i+8x)}{27\left(3ix+4x^2\right)^{3/2}}+\frac{32}{27}\int \frac{1}{\left(3ix+4x^2\right)^{3/2}}\,dx\\ &= \frac{2(3i+8x)}{27\left(3ix+4x^2\right)^{3/2}}+\frac{64(3i+8x)}{243\sqrt{3ix+4x^2}}\end{aligned}$$

Mathematica [A] time = 0.0106478, size = 36, normalized size = 0.68

$$\frac{2048x^3+2304ix^2-432x+54i}{243(x(4x+3i))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(-5/2),x]
```

```
[Out] (54*I - 432*x + (2304*I)*x^2 + 2048*x^3)/(243*(x*(3*I + 4*x))^(3/2))
```

Maple [A] time = 0.098, size = 42, normalized size = 0.8

$$\frac{6i+16x}{27}\left(3ix+4x^2\right)^{-\frac{3}{2}}+\frac{192i+512x}{243}\frac{1}{\sqrt{3ix+4x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*I*x+4*x^2)^(5/2),x)
```

```
[Out] 2/27*(3*I+8*x)/(3*I*x+4*x^2)^(3/2)+64/243*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

Maxima [A] time = 1.20834, size = 74, normalized size = 1.4

$$\frac{512x}{243\sqrt{4x^2+3ix}}+\frac{64i}{81\sqrt{4x^2+3ix}}+\frac{16x}{27\left(4x^2+3ix\right)^{\frac{3}{2}}}+\frac{2i}{9\left(4x^2+3ix\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 512/243*x/sqrt(4*x^2 + 3*I*x) + 64/81*I/sqrt(4*x^2 + 3*I*x) + 16/27*x/(4*x^
2 + 3*I*x)^(3/2) + 2/9*I/(4*x^2 + 3*I*x)^(3/2)
```

Fricas [A] time = 2.51191, size = 178, normalized size = 3.36

$$\frac{4096x^4+6144ix^3-2304x^2+\left(2048x^3+2304ix^2-432x+54i\right)\sqrt{4x^2+3ix}}{3888x^4+5832ix^3-2187x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (4096*x^4 + 6144*I*x^3 - 2304*x^2 + (2048*x^3 + 2304*I*x^2 - 432*x + 54*I)*
sqrt(4*x^2 + 3*I*x))/(3888*x^4 + 5832*I*x^3 - 2187*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(4x^2+3ix\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x**2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(-5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.20 $\int \frac{1}{\left(3ix+4x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=79

$$\frac{4096(8x+3i)}{10935\sqrt{4x^2+3ix}}+\frac{128(8x+3i)}{1215\left(4x^2+3ix\right)^{3/2}}+\frac{2(8x+3i)}{45\left(4x^2+3ix\right)^{5/2}}$$

```
[Out] (2*(3*I + 8*x))/(45*((3*I)*x + 4*x^2)^(5/2)) + (128*(3*I + 8*x))/(1215*((3*
I)*x + 4*x^2)^(3/2)) + (4096*(3*I + 8*x))/(10935*Sqrt[(3*I)*x + 4*x^2])
```

Rubi [A] time = 0.0124143, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {614, 613}

$$\frac{4096(8x+3i)}{10935\sqrt{4x^2+3ix}}+\frac{128(8x+3i)}{1215\left(4x^2+3ix\right)^{3/2}}+\frac{2(8x+3i)}{45\left(4x^2+3ix\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(-7/2),x]
```

```
[Out] (2*(3*I + 8*x))/(45*((3*I)*x + 4*x^2)^(5/2)) + (128*(3*I + 8*x))/(1215*((3*
I)*x + 4*x^2)^(3/2)) + (4096*(3*I + 8*x))/(10935*Sqrt[(3*I)*x + 4*x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(3ix+4x^2\right)^{7/2}}\,dx &= \frac{2(3i+8x)}{45\left(3ix+4x^2\right)^{5/2}}+\frac{64}{45}\int \frac{1}{\left(3ix+4x^2\right)^{5/2}}\,dx\\ &= \frac{2(3i+8x)}{45\left(3ix+4x^2\right)^{5/2}}+\frac{128(3i+8x)}{1215\left(3ix+4x^2\right)^{3/2}}+\frac{2048\int \frac{1}{\left(3ix+4x^2\right)^{3/2}}\,dx}{1215}\\ &= \frac{2(3i+8x)}{45\left(3ix+4x^2\right)^{5/2}}+\frac{128(3i+8x)}{1215\left(3ix+4x^2\right)^{3/2}}+\frac{4096(3i+8x)}{10935\sqrt{3ix+4x^2}}\end{aligned}$$

Mathematica [A] time = 0.0148155, size = 48, normalized size = 0.61

$$\frac{524288x^5+983040ix^4-552960x^3-69120ix^2-6480x+1458i}{10935(x(4x+3i))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(-7/2),x]
```

```
[Out] (1458*I - 6480*x - (69120*I)*x^2 - 552960*x^3 + (983040*I)*x^4 + 524288*x^5
)/(10935*(x*(3*I + 4*x))^(5/2))
```

Maple [A] time = 0.108, size = 62, normalized size = 0.8

$$\frac{6i+16x}{45}\left(3ix+4x^2\right)^{-\frac{5}{2}}+\frac{384i+1024x}{1215}\left(3ix+4x^2\right)^{-\frac{3}{2}}+\frac{12288i+32768x}{10935}\frac{1}{\sqrt{3ix+4x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*I*x+4*x^2)^(7/2),x)
```

```
[Out] 2/45*(3*I+8*x)/(3*I*x+4*x^2)^(5/2)+128/1215*(3*I+8*x)/(3*I*x+4*x^2)^(3/2)+4
096/10935*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

Maxima [A] time = 1.37852, size = 111, normalized size = 1.41

$$\frac{32768x}{10935\sqrt{4x^2+3ix}}+\frac{4096i}{3645\sqrt{4x^2+3ix}}+\frac{1024x}{1215\left(4x^2+3ix\right)^{\frac{3}{2}}}+\frac{128i}{405\left(4x^2+3ix\right)^{\frac{3}{2}}}+\frac{16x}{45\left(4x^2+3ix\right)^{\frac{5}{2}}}+\frac{2i}{15\left(4x^2+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 32768/10935*x/sqrt(4*x^2 + 3*I*x) + 4096/3645*I/sqrt(4*x^2 + 3*I*x) + 1024/
1215*x/(4*x^2 + 3*I*x)^(3/2) + 128/405*I/(4*x^2 + 3*I*x)^(3/2) + 16/45*x/(4
*x^2 + 3*I*x)^(5/2) + 2/15*I/(4*x^2 + 3*I*x)^(5/2)
```

Fricas [A] time = 2.41308, size = 288, normalized size = 3.65

$$\frac{1048576x^6+2359296ix^5-1769472x^4-442368ix^3+\left(524288x^5+983040ix^4-552960x^3-69120ix^2-6480x+\right.}{699840x^6+1574640ix^5-1180980x^4-295245ix^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] (1048576*x^6 + 2359296*I*x^5 - 1769472*x^4 - 442368*I*x^3 + (524288*x^5 + 9
83040*I*x^4 - 552960*x^3 - 69120*I*x^2 - 6480*x + 1458*I)*sqrt(4*x^2 + 3*I*
x))/(699840*x^6 + 1574640*I*x^5 - 1180980*x^4 - 295245*I*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(4x^2+3ix\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x**2)**(7/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(-7/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.21 $\int \frac{1}{\sqrt{3x-4x^2}} \, dx$

Optimal. Leaf size=12

$$-\frac{1}{2}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

```
[Out] -ArcSin[1 - (8*x)/3]/2
```

Rubi [A] time = 0.0056575, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {619, 216}

$$-\frac{1}{2}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3*x - 4*x^2],x]

[Out] -ArcSin[1 - (8*x)/3]/2
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{3x-4x^2}} \, dx &= -\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}} \, dx, x, 3-8x\right)\right) \\ &= -\frac{1}{2}\sin^{-1}\left(1-\frac{8x}{3}\right) \end{aligned}$$

Mathematica [B] time = 0.0138426, size = 40, normalized size = 3.33

$$-\frac{\sqrt{-x(4x-3)}\sin^{-1}\left(\sqrt{1-\frac{4x}{3}}\right)}{\sqrt{3-4x}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3*x - 4*x^2],x]
```

```
[Out] -((Sqrt[-(x*(-3 + 4*x))]*ArcSin[Sqrt[1 - (4*x)/3]])/(Sqrt[3 - 4*x]*Sqrt[x])
)
```

Maple [A] time = 0.048, size = 9, normalized size = 0.8

$$\frac{1}{2}\arcsin\left(-1+\frac{8x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+3*x)^(1/2),x)
```

```
[Out] 1/2*arcsin(-1+8/3*x)
```

Maxima [A] time = 1.78698, size = 11, normalized size = 0.92

$$-\frac{1}{2}\arcsin\left(-\frac{8}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(-8/3*x + 1)
```

Fricas [B] time = 2.33111, size = 47, normalized size = 3.92

$$-\arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-4x^2+3x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-4*x**2 + 3*x), x)
```

Giac [A] time = 1.2208, size = 11, normalized size = 0.92

$$\frac{1}{2}\arcsin\left(\frac{8}{3}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(8/3*x - 1)
```

3.22 $\int \frac{1}{\left(3x-4x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

```
[Out] (-2*(3 - 8*x))/(9*Sqrt[3*x - 4*x^2])
```

Rubi [A] time = 0.0023807, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {613}

$$-\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(-3/2),x]
```

```
[Out] (-2*(3 - 8*x))/(9*Sqrt[3*x - 4*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(3x-4x^2\right)^{3/2}}\,dx=-\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

Mathematica [A] time = 0.0057725, size = 21, normalized size = 0.95

$$\frac{2(8x-3)}{9\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(-3/2),x]
```

```
[Out] (2*(-3 + 8*x))/(9*Sqrt[-(x*(-3 + 4*x))])
```

Maple [A] time = 0.047, size = 25, normalized size = 1.1

$$-\frac{2\,x\,(-3+4\,x)\,(-3+8\,x)}{9}\left(-4\,x^2+3\,x\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+3*x)^(3/2),x)
```

```
[Out] -2/9*x*(-3+4*x)*(-3+8*x)/(-4*x^2+3*x)^(3/2)
```

Maxima [A] time = 1.16826, size = 38, normalized size = 1.73

$$\frac{16\,x}{9\,\sqrt{-4\,x^2+3\,x}}-\frac{2}{3\,\sqrt{-4\,x^2+3\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 16/9*x/sqrt(-4*x^2 + 3*x) - 2/3/sqrt(-4*x^2 + 3*x)
```

Fricas [A] time = 2.16481, size = 66, normalized size = 3.

$$-\frac{2\,\sqrt{-4\,x^2+3\,x}(8\,x-3)}{9\left(4\,x^2-3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/9*sqrt(-4*x^2 + 3*x)*(8*x - 3)/(4*x^2 - 3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-4x^2+3x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(3/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(-3/2), x)
```

Giac [A] time = 1.32198, size = 39, normalized size = 1.77

$$-\frac{2\,\sqrt{-4\,x^2+3\,x}(8\,x-3)}{9\left(4\,x^2-3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/9*sqrt(-4*x^2 + 3*x)*(8*x - 3)/(4*x^2 - 3*x)
```

3.23 $\int \frac{1}{\left(3x-4x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}-\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}$$

```
[Out] (-2*(3 - 8*x))/(27*(3*x - 4*x^2)^(3/2)) - (64*(3 - 8*x))/(243*Sqrt[3*x - 4*
x^2])
```

Rubi [A] time = 0.0062287, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {614, 613}

$$-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}-\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(-5/2),x]
```

```
[Out] (-2*(3 - 8*x))/(27*(3*x - 4*x^2)^(3/2)) - (64*(3 - 8*x))/(243*Sqrt[3*x - 4*
x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(3x-4x^2\right)^{5/2}}\,dx &= -\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}+\frac{32}{27}\int \frac{1}{\left(3x-4x^2\right)^{3/2}}\,dx\\ &= -\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}\end{aligned}$$

Mathematica [A] time = 0.0101693, size = 31, normalized size = 0.69

$$-\frac{2048x^3-2304x^2+432x+54}{243(-x(4x-3))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(-5/2),x]
```

```
[Out] -(54 + 432*x - 2304*x^2 + 2048*x^3)/(243*(-(x*(-3 + 4*x)))^(3/2))
```

Maple [A] time = 0.048, size = 35, normalized size = 0.8

$$\frac{2\,x\,(-3+4\,x)\left(1024\,x^{3}-1152\,x^{2}+216\,x+27\right)}{243}\left(-4\,x^{2}+3\,x\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+3*x)^(5/2),x)
```

```
[Out] 2/243*x*(-3+4*x)*(1024*x^3-1152*x^2+216*x+27)/(-4*x^2+3*x)^(5/2)
```

Maxima [A] time = 1.2355, size = 74, normalized size = 1.64

$$\frac{512\,x}{243\,\sqrt{-4\,x^{2}+3\,x}}-\frac{64}{81\,\sqrt{-4\,x^{2}+3\,x}}+\frac{16\,x}{27\left(-4\,x^{2}+3\,x\right)^{\frac{3}{2}}}-\frac{2}{9\left(-4\,x^{2}+3\,x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 512/243*x/sqrt(-4*x^2 + 3*x) - 64/81/sqrt(-4*x^2 + 3*x) + 16/27*x/(-4*x^2 +
 3*x)^(3/2) - 2/9/(-4*x^2 + 3*x)^(3/2)
```

Fricas [A] time = 2.19676, size = 119, normalized size = 2.64

$$-\frac{2\left(1024\,x^{3}-1152\,x^{2}+216\,x+27\right)\sqrt{-4\,x^{2}+3\,x}}{243\left(16\,x^{4}-24\,x^{3}+9\,x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/243*(1024*x^3 - 1152*x^2 + 216*x + 27)*sqrt(-4*x^2 + 3*x)/(16*x^4 - 24*x
^3 + 9*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-4x^{2}+3x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(5/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(-5/2), x)
```

Giac [A] time = 1.35491, size = 53, normalized size = 1.18

$$-\frac{2\,(8\,(16\,(8\,x-9)x+27)x+27)\sqrt{-4\,x^2+3\,x}}{243\,\left(4\,x^2-3\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/243*(8*(16*(8*x - 9)*x + 27)*x + 27)*sqrt(-4*x^2 + 3*x)/(4*x^2 - 3*x)^2
```

3.24 $\int \frac{1}{(3x-4x^2)^{7/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{4096(3-8x)}{10935\sqrt{3x-4x^2}}-\frac{128(3-8x)}{1215\left(3x-4x^2\right)^{3/2}}-\frac{2(3-8x)}{45\left(3x-4x^2\right)^{5/2}}$$

```
[Out] (-2*(3 - 8*x))/(45*(3*x - 4*x^2)^(5/2)) - (128*(3 - 8*x))/(1215*(3*x - 4*x^
2)^(3/2)) - (4096*(3 - 8*x))/(10935*Sqrt[3*x - 4*x^2])
```

Rubi [A] time = 0.0115283, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {614, 613}

$$-\frac{4096(3-8x)}{10935\sqrt{3x-4x^2}}-\frac{128(3-8x)}{1215\left(3x-4x^2\right)^{3/2}}-\frac{2(3-8x)}{45\left(3x-4x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(-7/2),x]
```

```
[Out] (-2*(3 - 8*x))/(45*(3*x - 4*x^2)^(5/2)) - (128*(3 - 8*x))/(1215*(3*x - 4*x^
2)^(3/2)) - (4096*(3 - 8*x))/(10935*Sqrt[3*x - 4*x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(3x-4x^2\right)^{7/2}}\,dx &= -\frac{2(3-8x)}{45\left(3x-4x^2\right)^{5/2}}+\frac{64}{45}\int \frac{1}{\left(3x-4x^2\right)^{5/2}}\,dx\\ &= -\frac{2(3-8x)}{45\left(3x-4x^2\right)^{5/2}}-\frac{128(3-8x)}{1215\left(3x-4x^2\right)^{3/2}}+\frac{2048\int \frac{1}{(3x-4x^2)^{3/2}}\,dx}{1215}\\ &= -\frac{2(3-8x)}{45\left(3x-4x^2\right)^{5/2}}-\frac{128(3-8x)}{1215\left(3x-4x^2\right)^{3/2}}-\frac{4096(3-8x)}{10935\sqrt{3x-4x^2}}\end{aligned}$$

Mathematica [A] time = 0.0148466, size = 51, normalized size = 0.76

$$\frac{2\left(262144x^5-491520x^4+276480x^3-34560x^2-3240x-729\right)}{10935(3-4x)^2x^2\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(-7/2),x]
```

```
[Out] (2*(-729 - 3240*x - 34560*x^2 + 276480*x^3 - 491520*x^4 + 262144*x^5))/(109
35*(3 - 4*x)^2*x^2*Sqrt[-(x*(-3 + 4*x))])
```

Maple [A] time = 0.048, size = 45, normalized size = 0.7

$$-\frac{2x(-3+4x)\left(262144x^5-491520x^4+276480x^3-34560x^2-3240x-729\right)}{10935}\left(-4x^2+3x\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+3*x)^(7/2),x)
```

```
[Out] -2/10935*x*(-3+4*x)*(262144*x^5-491520*x^4+276480*x^3-34560*x^2-3240*x-729)
/(-4*x^2+3*x)^(7/2)
```

Maxima [A] time = 1.36279, size = 111, normalized size = 1.66

$$\frac{32768x}{10935\sqrt{-4x^2+3x}}-\frac{4096}{3645\sqrt{-4x^2+3x}}+\frac{1024x}{1215\left(-4x^2+3x\right)^{\frac{3}{2}}}-\frac{128}{405\left(-4x^2+3x\right)^{\frac{3}{2}}}+\frac{16x}{45\left(-4x^2+3x\right)^{\frac{5}{2}}}-\frac{2}{15\left(-4x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 32768/10935*x/sqrt(-4*x^2 + 3*x) - 4096/3645/sqrt(-4*x^2 + 3*x) + 1024/1215
*x/(-4*x^2 + 3*x)^(3/2) - 128/405/(-4*x^2 + 3*x)^(3/2) + 16/45*x/(-4*x^2 +
3*x)^(5/2) - 2/15/(-4*x^2 + 3*x)^(5/2)
```

Fricas [A] time = 2.24674, size = 180, normalized size = 2.69

$$-\frac{2\left(262144x^5-491520x^4+276480x^3-34560x^2-3240x-729\right)\sqrt{-4x^2+3x}}{10935\left(64x^6-144x^5+108x^4-27x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/10935*(262144*x^5 - 491520*x^4 + 276480*x^3 - 34560*x^2 - 3240*x - 729)*
sqrt(-4*x^2 + 3*x)/(64*x^6 - 144*x^5 + 108*x^4 - 27*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-4x^2+3x\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(7/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(-7/2), x)
```

Giac [A] time = 1.627, size = 66, normalized size = 0.99

$$-\frac{2\,(8\,(32\,(8\,(16\,(8\,x-15)x+135)x-135)x-405)x-729)\sqrt{-4\,x^2+3\,x}}{10935\,\left(4\,x^2-3\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/10935*(8*(32*(8*(16*(8*x - 15)*x + 135)*x - 135)*x - 405)*x - 729)*sqrt(
-4*x^2 + 3*x)/(4*x^2 - 3*x)^3
```

3.25 $\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin^{-1}(1-2bx)}{b}$$

[Out] -(ArcSin[1 - 2*b*x]/b)

Rubi [A] time = 0.0076931, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {619, 216}

$$-\frac{\sin^{-1}(1-2bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[b*x - b^2*x^2],x]

[Out] -(ArcSin[1 - 2*b*x]/b)

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{b^2}}}\,dx, x, b-2b^2x\right)}{b^2}\\ &= -\frac{\sin^{-1}(1-2bx)}{b}\end{aligned}$$

Mathematica [B] time = 0.014204, size = 47, normalized size = 3.92

$$\frac{2\sqrt{x}\sqrt{1-bx}\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\sqrt{-bx(bx-1)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[b*x - b^2*x^2],x]

```
[Out] (2*Sqrt[x]*Sqrt[1 - b*x]*ArcSin[Sqrt[b]*Sqrt[x]])/(Sqrt[b]*Sqrt[-(b*x*(-1 +
 b*x))])
```

Maple [B] time = 0.05, size = 35, normalized size = 2.9

$$\arctan\left(\sqrt{b^2}\left(x-\frac{1}{2b}\right)\frac{1}{\sqrt{-b^2x^2+bx}}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b^2*x^2+b*x)^(1/2),x)
```

```
[Out] 1/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x-1/2/b)/(-b^2*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.23184, size = 55, normalized size = 4.58

$$-\frac{2\arctan\left(\frac{\sqrt{-b^2x^2+bx}}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-b^2*x^2 + b*x)/(b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b**2*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-b**2*x**2 + b*x), x)
```

Giac [A] time = 2.63835, size = 20, normalized size = 1.67

$$-\frac{\arcsin\left(-2\,bx+1\right)\operatorname{sgn}\left(b\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(-2*b*x + 1)*sgn(b)/abs(b)
```

3.26 $\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx$

Optimal. Leaf size=24

$$\frac{2\tanh^{-1}\left(\frac{bx}{\sqrt{b^2x^2+bx}}\right)}{b}$$

```
[Out] (2*ArcTanh[(b*x)/Sqrt[b*x + b^2*x^2]])/b
```

Rubi [A] time = 0.0077753, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {620, 206}

$$\frac{2\tanh^{-1}\left(\frac{bx}{\sqrt{b^2x^2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x + b^2*x^2],x]
```

```
[Out] (2*ArcTanh[(b*x)/Sqrt[b*x + b^2*x^2]])/b
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-b^2x^2}\,dx, x, \frac{x}{\sqrt{bx+b^2x^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{bx}{\sqrt{bx+b^2x^2}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0203353, size = 45, normalized size = 1.88

$$\frac{2\sqrt{x}\sqrt{bx+1}\sinh^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\sqrt{bx(bx+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x + b^2*x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + b*x]*ArcSinh[Sqrt[b]*Sqrt[x]])/(Sqrt[b]*Sqrt[b*x*(1 + b
*x)])
```

Maple [A] time = 0.057, size = 37, normalized size = 1.5

$$\ln\left(\left(\frac{b}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+bx}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+b*x)^(1/2),x)
```

```
[Out] ln((1/2*b+b^2*x)/(b^2)^(1/2)+(b^2*x^2+b*x)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.15796, size = 59, normalized size = 2.46

$$-\frac{\log\left(-2\,bx+2\,\sqrt{b^2x^2+bx}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b^2*x^2 + b*x) - 1)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b^2x^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b**2*x**2 + b*x), x)
```

Giac [A] time = 1.80615, size = 49, normalized size = 2.04

$$-\frac{\log\left(\left|-2\left(x|b|-\sqrt{b^2x^2+bx}\right)|b|-b\right|\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(x*abs(b) - sqrt(b^2*x^2 + b*x))*abs(b) - b))/abs(b)
```

3.27 $\int \frac{1}{\sqrt{6x-x^2}}\,dx$

Optimal. Leaf size=10

$$-\sin^{-1}\left(1-\frac{x}{3}\right)$$

```
[Out] -ArcSin[1 - x/3]
```

Rubi [A] time = 0.0062504, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {619, 216}

$$-\sin^{-1}\left(1-\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[6*x - x^2],x]
```

```
[Out] -ArcSin[1 - x/3]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{6x-x^2}}\,dx &= -\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx, x, 6-2x\right)\right)\\ &= -\sin^{-1}\left(1-\frac{x}{3}\right) \end{aligned}$$

Mathematica [A] time = 0.0096672, size = 14, normalized size = 1.4

$$-2\sin^{-1}\left(\sqrt{1-\frac{x}{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[6*x - x^2],x]
```

```
[Out] -2*ArcSin[Sqrt[1 - x/6]]
```

Maple [A] time = 0.065, size = 7, normalized size = 0.7

$$\arcsin\left(-1+\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+6*x)^(1/2),x)
```

```
[Out] arcsin(-1+1/3*x)
```

Maxima [A] time = 3.88425, size = 11, normalized size = 1.1

$$-\arcsin\left(-\frac{1}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/3*x + 1)
```

Fricas [B] time = 2.27557, size = 42, normalized size = 4.2

$$-2\arctan\left(\frac{\sqrt{-x^2+6x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-x^2 + 6*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+6x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+6*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 + 6*x), x)
```

Giac [A] time = 1.61726, size = 8, normalized size = 0.8

$$\arcsin\left(\frac{1}{3}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/3*x - 1)
```

3.28 $\int \frac{1}{\sqrt{4x+x^2}}\,dx$

Optimal. Leaf size=16

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

```
[Out] 2*ArcTanh[x/Sqrt[4*x + x^2]]
```

Rubi [A] time = 0.0038626, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {620, 206}

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4*x + x^2],x]

[Out] 2*ArcTanh[x/Sqrt[4*x + x^2]]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{4x+x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{4x+x^2}}\right)\\ &= 2\tanh^{-1}\left(\frac{x}{\sqrt{4x+x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.0064175, size = 33, normalized size = 2.06

$$\frac{2\sqrt{x}\sqrt{x+4}\sinh^{-1}\left(\frac{\sqrt{x}}{2}\right)}{\sqrt{x(x+4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4*x + x^2],x]

[Out] (2*Sqrt[x]*Sqrt[4 + x]*ArcSinh[Sqrt[x]/2])/Sqrt[x*(4 + x)]
```

Maple [A] time = 0.052, size = 14, normalized size = 0.9

$$\ln\left(x+2+\sqrt{x^2+4x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4*x)^(1/2),x)
```

```
[Out] ln(x+2+(x^2+4*x)^(1/2))
```

Maxima [A] time = 2.49606, size = 23, normalized size = 1.44

$$\log\left(2x+2\sqrt{x^2+4x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 + 4*x) + 4)
```

Fricas [A] time = 2.28133, size = 43, normalized size = 2.69

$$-\log\left(-x+\sqrt{x^2+4x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 4*x) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+4x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2 + 4*x), x)
```

Giac [A] time = 1.85389, size = 24, normalized size = 1.5

$$-\log\left(\left|-x+\sqrt{x^2+4x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 + 4*x) - 2))
```

3.29 $\int \frac{1}{\sqrt{-2x+x^2}} \, dx$

Optimal. Leaf size=16

$$2 \tanh^{-1}\left(\frac{x}{\sqrt{x^2-2x}}\right)$$

```
[Out] 2*ArcTanh[x/Sqrt[-2*x + x^2]]
```

Rubi [A] time = 0.0040644, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {620, 206}

$$2 \tanh^{-1}\left(\frac{x}{\sqrt{x^2-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2*x + x^2],x]
```

```
[Out] 2*ArcTanh[x/Sqrt[-2*x + x^2]]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2x+x^2}} \, dx &= 2\,\text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, \frac{x}{\sqrt{-2x+x^2}}\right)\\ &= 2 \tanh^{-1}\left(\frac{x}{\sqrt{-2x+x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.0125127, size = 33, normalized size = 2.06

$$\frac{2\sqrt{(x-2)x}\sin^{-1}\left(\sqrt{1-\frac{x}{2}}\right)}{\sqrt{-(x-2)x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2*x + x^2],x]
```

```
[Out] (2*Sqrt[(-2 + x)*x]*ArcSin[Sqrt[1 - x/2]])/Sqrt[-((-2 + x)*x)]
```

Maple [A] time = 0.052, size = 14, normalized size = 0.9

$$\ln\left(-1+x+\sqrt{x^2-2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x)^(1/2),x)
```

```
[Out] ln(-1+x+(x^2-2*x)^(1/2))
```

Maxima [A] time = 1.1913, size = 23, normalized size = 1.44

$$\log\left(2x+2\sqrt{x^2-2x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 - 2*x) - 2)
```

Fricas [A] time = 2.47777, size = 43, normalized size = 2.69

$$-\log\left(-x+\sqrt{x^2-2x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 2*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2 - 2*x), x)
```

Giac [A] time = 1.26159, size = 24, normalized size = 1.5

$$-\log\left(\left|-x+\sqrt{x^2-2x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 - 2*x) + 1))
```

3.30 $\int \left(bx+cx^2\right)^{4/3}\,dx$

Optimal. Leaf size=448

$$\frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(bx+cx^2\right)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-}\right.\right.}{55c(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(1/3)*(b + 2*c*x)*(b*x + c*x^2)^(4/3))/(55*c*(-
((c*(b*x + c*x^2))/b^2))^(4/3)) + (3*(-((c*x*(b + c*x))/b^2))^(4/3)*(b + 2*
c*x)*(b*x + c*x^2)^(4/3))/(22*c*(-((c*(b*x + c*x^2))/b^2))^(4/3)) + (2^(1/3
)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(b*x + c*x^2)^(4/3)*(1 - 2^(2/3)*(-((c*x*(b
 + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*
2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b +
 c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b +
 c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))]
, -7 + 4*Sqrt[3]])/(55*c*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(4/3)*Sqrt[
-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c
*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 0.716936, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {622, 619, 195, 236, 219}

$$\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}}+\frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\left(bx+cx^2\right)^{4/3}}{55c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}}+\frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(bx+cx^2\right)^{4/3}\left(1-2^{2}\right.}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(4/3),x]
```

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(1/3)*(b + 2*c*x)*(b*x + c*x^2)^(4/3))/(55*c*(-
((c*(b*x + c*x^2))/b^2))^(4/3)) + (3*(-((c*x*(b + c*x))/b^2))^(4/3)*(b + 2*
c*x)*(b*x + c*x^2)^(4/3))/(22*c*(-((c*(b*x + c*x^2))/b^2))^(4/3)) + (2^(1/3
)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(b*x + c*x^2)^(4/3)*(1 - 2^(2/3)*(-((c*x*(b
 + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*
2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b +
 c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b +
 c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))]
, -7 + 4*Sqrt[3]])/(55*c*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(4/3)*Sqrt[
-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c
*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
```

```
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \left(bx+cx^2\right)^{4/3} dx = \frac{\left(bx+cx^2\right)^{4/3} \int \left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{4/3} dx}{\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}}$$

$$= -\frac{\left(b^2\left(bx+cx^2\right)^{4/3}\right) \text{Subst}\left(\int \left(1-\frac{b^2x^2}{c^2}\right)^{4/3} dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{8\ 2^{2/3}c^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}}$$

$$= \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}} - \frac{\left(b^2\left(bx+cx^2\right)^{4/3}\right) \text{Subst}\left(\int \sqrt[3]{1-\frac{b^2x^2}{c^2}} dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right.}{11\ 2^{2/3}c^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}}$$

$$= \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\left(bx+cx^2\right)^{4/3}}{55c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}} - \frac{\left(\sqrt[3]{2}b^2\left(bx+cx^2\right.\right.}{}$$

$$= \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\left(bx+cx^2\right)^{4/3}}{55c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}} + \frac{\left(3\left(bx+cx^2\right)^{4/3}\right.}{}$$

$$= \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\left(bx+cx^2\right)^{4/3}}{55c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}} + \frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}}{}$$

Mathematica [C] time = 0.016001, size = 48, normalized size = 0.11

$$\frac{3bx^2\sqrt[3]{x(b+cx)}\,{}_2F_1\left(-\frac{4}{3},\frac{7}{3};\frac{10}{3};-\frac{cx}{b}\right)}{7\sqrt[3]{\frac{cx}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(4/3),x]
```

```
[Out] (3*b*x^2*(x*(b + c*x))^(1/3)*Hypergeometric2F1[-4/3, 7/3, 10/3, -((c*x)/b)]
)/(7*(1 + (c*x)/b)^(1/3))
```

Maple [F] time = 0.456, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(4/3),x)
```

```
[Out] int((c*x^2+b*x)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{4}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(4/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx + cx^2\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(4/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(4/3), x)
```

3.31 $\int \sqrt[3]{bx+cx^2}\,dx$

Optimal. Leaf size=387

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\sqrt[3]{bx+cx^2}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\right.}{5\,2^{2/3}c(b+2cx)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(1/3)*(b + 2*c*x)*(b*x + c*x^2)^(1/3))/(10*c*(-
((c*(b*x + c*x^2))/b^2))^(1/3)) + (3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(b*x + c*x
^2)^(1/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1
 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)
*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*2^(2/3)*c*(b + 2*c*x
)*(-((c*(b*x + c*x^2))/b^2))^(1/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b
^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 0.449161, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {622, 619, 195, 236, 219}

$$\frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}+\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\sqrt[3]{bx+cx^2}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\ldots\right.}{5\,2^{2/3}c(b+2cx)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+c\ldots}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\ldots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(1/3),x]
```

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(1/3)*(b + 2*c*x)*(b*x + c*x^2)^(1/3))/(10*c*(-
((c*(b*x + c*x^2))/b^2))^(1/3)) + (3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(b*x + c*x
^2)^(1/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1
 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)
*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*2^(2/3)*c*(b + 2*c*x
)*(-((c*(b*x + c*x^2))/b^2))^(1/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b
^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \sqrt[3]{bx+cx^2}\,dx = \frac{\sqrt[3]{bx+cx^2}\int \sqrt[3]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}\,dx}{\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}$$

$$= -\frac{\left(b^2\sqrt[3]{bx+cx^2}\right)\text{Subst}\left(\int \sqrt[3]{1-\frac{b^2x^2}{c^2}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{2\;2^{2/3}c^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}$$

$$= \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} - \frac{\left(b^2\sqrt[3]{bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{2/3}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{5\;2^{2/3}c^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}$$

$$= \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} + \frac{\left(3\sqrt[3]{bx+cx^2}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, 2^{2/3}\sqrt[3]{-\frac{cx(1\cdots}{}}\right.}{10\;2^{2/3}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}$$

$$= \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\sqrt[3]{bx+cx^2}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\cdots}{5\;2^{2/3}c(b+2cx)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-\cdots}}$$

Mathematica [C] time = 0.0103477, size = 45, normalized size = 0.12

$$\frac{3x\sqrt[3]{x(b+cx)}\,{}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{7}{3};-\frac{cx}{b}\right)}{4\sqrt[3]{\frac{cx}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(1/3),x]
```

```
[Out] (3*x*(x*(b + c*x))^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, -((c*x)/b)])/(4*
(1 + (c*x)/b)^(1/3))
```

Maple [F] time = 0.435, size = 0, normalized size = 0.

$$\int \sqrt[3]{cx^2+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/3),x)
```

```
[Out] int((c*x^2+b*x)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^2+bx\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(1/3), x)
```

3.32 $\int \frac{1}{\left(bx+cx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=322

$$\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}\right.\right.}{c(b+2cx)\left(bx+cx^2\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(2/3)*(1
- 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*
x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] -
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b +
 c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)^(2/3)*S
qrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(
-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 0.394286, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {622, 619, 236, 219}

$$\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\right|}{c(b+2cx)\left(bx+cx^2\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-2/3),x]
```

```
[Out] (2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(2/3)*(1
- 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*
x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] -
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b +
 c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)^(2/3)*S
qrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(
-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(bx+cx^2\right)^{2/3}}\,dx &= \frac{\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{2/3}}\,dx}{\left(bx+cx^2\right)^{2/3}} \\
&= -\frac{\left(\sqrt[3]{2}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{2/3}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\left(bx+cx^2\right)^{2/3}} \\
&= \frac{\left(3\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, 2^{2/3}\sqrt[3]{-\frac{cx\left(1+\frac{cx}{b}\right)}{b}}\right)}{2^{2/3}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)\left(bx+cx^2\right)^{2/3}} \\
&= \frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx(b}{b}\cdots\right.}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\cdots\right.}}\cdots}{c(b+2cx)\left(bx+cx^2\right)^{2/3}\sqrt{-1-\frac{4cx(b+cx)}{b^2}}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(}{}}\cdots}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{c}{}}\cdots\right.}}\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.0112151, size = 43, normalized size = 0.13

$$
\frac{3x\left(\frac{cx}{b}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{cx}{b}\right)}{(x(b+cx))^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-2/3),x]
```

```
[Out] (3*x*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((c*x)/b)])/(x*(
b + c*x))^(2/3)
```

Maple [F] time = 0.651, size = 0, normalized size = 0.

$$\int \left(c x^2+b x\right)^{-\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(2/3),x)
```

```
[Out] int(1/(c*x^2+b*x)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c x^2+b x\right)^{\frac{2}{3}}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(c x^2+b x\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(-2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x+c x^2\right)^{\frac{2}{3}}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(2/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c x^2+b x\right)^{\frac{2}{3}}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-2/3), x)
```

3.33 $\int \frac{1}{\left(bx+cx^2\right)^{5/3}}\,dx$

Optimal. Leaf size=384

$$\frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{c(b+2cx)\left(bx+cx^2\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(5/3))/(2*c*(-((c*x*(b + c*x))/b^
2))^(2/3)*(b*x + c*x^2)^(5/3)) + (2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(-(
(c*(b*x + c*x^2))/b^2))^(5/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*
Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c
*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]
*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*
(b + 2*c*x)*(b*x + c*x^2)^(5/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2)
)^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 0.457139, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {622, 619, 199, 236, 219}

$$\frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}\left(bx+cx^2\right)^{5/3}}+\frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{c(b+2cx)\left(bx+cx^2\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-5/3),x]
```

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(5/3))/(2*c*(-((c*x*(b + c*x))/b^
2))^(2/3)*(b*x + c*x^2)^(5/3)) + (2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(-(
(c*(b*x + c*x^2))/b^2))^(5/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*
Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c
*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]
*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*
(b + 2*c*x)*(b*x + c*x^2)^(5/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2)
)^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{\left(bx+cx^2\right)^{5/3}}\,dx = \frac{\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{5/3}}\,dx}{\left(bx+cx^2\right)^{5/3}}$$

$$= -\frac{\left(4\sqrt[3]{2}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{5/3}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\left(bx+cx^2\right)^{5/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}\left(bx+cx^2\right)^{5/3}} - \frac{\left(\sqrt[3]{2}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{2/3}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\left(bx+cx^2\right)^{5/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}\left(bx+cx^2\right)^{5/3}} + \frac{\left(3\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, 2^{2/3}\sqrt[3]{\cdots}\right.}{2^{2/3}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)\left(bx+cx^2\right)^{5/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}\left(bx+cx^2\right)^{5/3}} + \frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx}{\cdots}}\right.}{c(b+2cx)\left(bx+cx^2\right)^{5/3}\cdots}$$

Mathematica [C] time = 0.0120089, size = 47, normalized size = 0.12

$$-\frac{3\left(\frac{cx}{b}+1\right)^{2/3}{}_2F_1\left(-\frac{2}{3},\frac{5}{3};\frac{1}{3};-\frac{cx}{b}\right)}{2b(x(b+cx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-5/3),x]
```

```
[Out] (-3*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[-2/3, 5/3, 1/3, -((c*x)/b)])/(2*b
*(x*(b + c*x))^(2/3))
```

Maple [F] time = 0.613, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(5/3),x)
```

```
[Out] int(1/(c*x^2+b*x)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cx^2+bx\right)^{\frac{1}{3}}}{c^2x^4+2\,bcx^3+b^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(1/3)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx + cx^2\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(5/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + bx\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-5/3), x)
```

3.34 $\int \frac{1}{\left(bx+cx^2\right)^{8/3}}\,dx$

Optimal. Leaf size=448

$$\frac{14\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}\text{EllipticF}\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b}{b}}}{-2^{2/3}\sqrt[3]{-\frac{cx(b}{b}}}\right.\right.}{5c(b+2cx)\left(bx+cx^2\right)^{8/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(8/3))/(5*c*(-((c*x*(b + c*x))/b^
2))^(5/3)*(b*x + c*x^2)^(8/3)) + (21*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))
^(8/3))/(5*c*(-((c*x*(b + c*x))/b^2))^(2/3)*(b*x + c*x^2)^(8/3)) + (14*2^(1
/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(8/3)*(1 - 2^(
2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b
^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/
3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/
3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x)
)/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*c*(b + 2*c*x)*(b*x + c*x^2)^(8/3)*Sqrt
[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 0.509723, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {622, 619, 199, 236, 219}

$$\frac{21(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}\left(bx+cx^2\right)^{8/3}}+\frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}\left(bx+cx^2\right)^{8/3}}+\frac{14\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b}{b}\right.}{\left(-2^{2/3}\right.}}}{5c(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-8/3),x]
```

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(8/3))/(5*c*(-((c*x*(b + c*x))/b^
2))^(5/3)*(b*x + c*x^2)^(8/3)) + (21*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))
^(8/3))/(5*c*(-((c*x*(b + c*x))/b^2))^(2/3)*(b*x + c*x^2)^(8/3)) + (14*2^(1
/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(8/3)*(1 - 2^(
2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b
^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/
3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/
3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x)
)/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*c*(b + 2*c*x)*(b*x + c*x^2)^(8/3)*Sqrt
[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
```

```
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{\left(bx+cx^2\right)^{8/3}}\,dx=\frac{\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{8/3}}\,dx}{\left(bx+cx^2\right)^{8/3}}$$

$$=-\frac{\left(16\sqrt[3]{2}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{8/3}}\,dx,x,-\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\left(bx+cx^2\right)^{8/3}}$$

$$=\frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}\left(bx+cx^2\right)^{8/3}}-\frac{\left(56\sqrt[3]{2}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{5/3}}\,dx,x,-\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{5c^2\left(bx+cx^2\right)^{8/3}}$$

$$=\frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}\left(bx+cx^2\right)^{8/3}}+\frac{21(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}\left(bx+cx^2\right)^{8/3}}-\frac{\left(14\sqrt[3]{2}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}\right)\text{Subs}}{5c^2\left(bx\right.}$$

$$=\frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}\left(bx+cx^2\right)^{8/3}}+\frac{21(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}\left(bx+cx^2\right)^{8/3}}+\frac{\left(21\sqrt[3]{2}\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}\sqrt{-1-\ldots}\right.}{5\left(\ldots\right.}$$

$$=\frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}\left(bx+cx^2\right)^{8/3}}+\frac{21(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}\left(bx+cx^2\right)^{8/3}}+\frac{14\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(-\frac{c\left(bx+cx\ldots\right.}{b^2}\right.}{\ldots}$$

Mathematica [C] time = 0.0129575, size = 50, normalized size = 0.11

$$-\frac{3\left(\frac{cx}{b}+1\right)^{2/3}{}_2F_1\left(-\frac{5}{3},\frac{8}{3};-\frac{2}{3};-\frac{cx}{b}\right)}{5b^2x(x(b+cx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-8/3),x]
```

```
[Out] (-3*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[-5/3, 8/3, -2/3, -((c*x)/b)])/(5*
b^2*x*(x*(b + c*x))^(2/3))
```

Maple [F] time = 1.226, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{-\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(8/3),x)
```

```
[Out] int(1/(c*x^2+b*x)^(8/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-8/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx\right)^{\frac{1}{3}}}{c^3x^6+3\,bc^2x^5+3\,b^2cx^4+b^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(1/3)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx+cx^2\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(8/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-8/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-8/3), x)
```

3.35 $\int \left(bx+cx^2\right)^{5/3}\,dx$

Optimal. Leaf size=842

$$-\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(cx^2+bx\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\right|-\cdots}{364\sqrt[3]{2}c(b+2cx)\left(-\frac{c\left(cx^2+bx\right)}{b^2}\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (15*(-((c*x*(b + c*x))/b^2))^(2/3)*(b + 2*c*x)*(b*x + c*x^2)^(5/3))/(364*c*
(-((c*(b*x + c*x^2))/b^2))^(5/3)) + (3*(-((c*x*(b + c*x))/b^2))^(5/3)*(b +
2*c*x)*(b*x + c*x^2)^(5/3))/(26*c*(-((c*(b*x + c*x^2))/b^2))^(5/3)) - (15*(
b + 2*c*x)*(b*x + c*x^2)^(5/3))/(182*2^(1/3)*c*(-((c*(b*x + c*x^2))/b^2))^(
5/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))) - (15*3^(1/4)*
Sqrt[2 + Sqrt[3]]*b^2*(b*x + c*x^2)^(5/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b
^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-
((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2
))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2
))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*S
qrt[3]])/(364*2^(1/3)*c*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(5/3)*Sqrt[-
((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*
x*(b + c*x))/b^2))^(1/3))^2]) + (5*3^(3/4)*b^2*(b*x + c*x^2)^(5/3)*(1 - 2^
(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/
b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2
/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2
/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(91*2^(5/6)*c*(b + 2*c*x)*(-((c*(b*x + c
*x^2))/b^2))^(5/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 -
 Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 1.05795, antiderivative size = 842, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {622, 619, 195, 235, 304, 219, 1879}

$$-\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(cx^2+bx\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\right|-\cdots}{364\sqrt[3]{2}c(b+2cx)\left(-\frac{c\left(cx^2+bx\right)}{b^2}\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/3),x]
```


```
[Out] (15*(-((c*x*(b + c*x))/b^2))^(2/3)*(b + 2*c*x)*(b*x + c*x^2)^(5/3))/(364*c*
(-((c*(b*x + c*x^2))/b^2))^(5/3)) + (3*(-((c*x*(b + c*x))/b^2))^(5/3)*(b +
2*c*x)*(b*x + c*x^2)^(5/3))/(26*c*(-((c*(b*x + c*x^2))/b^2))^(5/3)) - (15*(
b + 2*c*x)*(b*x + c*x^2)^(5/3))/(182*2^(1/3)*c*(-((c*(b*x + c*x^2))/b^2))^(
5/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))) - (15*3^(1/4)*
Sqrt[2 + Sqrt[3]]*b^2*(b*x + c*x^2)^(5/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b
^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-
```

```
((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2
))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2
))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*S
qrt[3]])/(364*2^(1/3)*c*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(5/3)*Sqrt[-
((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*
x*(b + c*x))/b^2))^(1/3))^2)]) + (5*3^(3/4)*b^2*(b*x + c*x^2)^(5/3)*(1 - 2^
(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/
b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2
/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2
/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(91*2^(5/6)*c*(b + 2*c*x)*(-((c*(b*x + c
*x^2))/b^2))^(5/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 -
 Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
```

```
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(bx+cx^2\right)^{5/3}\,dx = \frac{\left(bx+cx^2\right)^{5/3}\int \left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{5/3}dx}{\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}}$$

$$= -\frac{\left(b^2\left(bx+cx^2\right)^{5/3}\right)\text{Subst}\left(\int \left(1-\frac{b^2x^2}{c^2}\right)^{5/3}dx,x,-\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{16\sqrt[3]{2}c^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}}$$

$$= \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}} - \frac{\left(5b^2\left(bx+cx^2\right)^{5/3}\right)\text{Subst}\left(\int \left(1-\frac{b^2x^2}{c^2}\right)^{2/3}dx,x,-\frac{c}{b}-\ldots}{104\sqrt[3]{2}c^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}}$$

$$= \frac{15\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{5/3}}{364c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}} - \frac{\left(5b^2\left(bx+c\ldots\right.\right.}{\ldots}$$

$$= \frac{15\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{5/3}}{364c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}} + \frac{\left(15\left(bx+cx\ldots\right.\right.}{\ldots}$$

$$= \frac{15\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{5/3}}{364c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}} - \frac{\left(15\left(bx+cx\ldots\right.\right.}{\ldots}$$

$$= \frac{15\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{5/3}}{364c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{5/3}} + \frac{15b^2\left(bx\ldots\right.}{182\sqrt[3]{2}c(b+\ldots}$$

Mathematica [C] time = 0.0138841, size = 48, normalized size = 0.06

$$\frac{3bx^2(x(b+cx))^{2/3}\,{}_2F_1\left(-\frac{5}{3},\frac{8}{3};\frac{11}{3};-\frac{cx}{b}\right)}{8\left(\frac{cx}{b}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/3),x]
```

```
[Out] (3*b*x^2*(x*(b + c*x))^(2/3)*Hypergeometric2F1[-5/3, 8/3, 11/3, -((c*x)/b)]
)/(8*(1 + (c*x)/b)^(2/3))
```

Maple [F] time = 0.574, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/3),x)
```

```
[Out] int((c*x^2+b*x)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx\right)^{\frac{5}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(5/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx+cx^2\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(5/3), x)
```

3.36 $\int \left(bx + cx^2\right)^{2/3} dx$

Optimal. Leaf size=781

$$\frac{\sqrt[6]{2}3^{3/4}b^2\left(bx+cx^2\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right),4\sqrt{3}\right.}{7c(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(2/3)*(b + 2*c*x)*(b*x + c*x^2)^(2/3))/(14*c*(-
((c*(b*x + c*x^2))/b^2))^(2/3)) - (3*(b + 2*c*x)*(b*x + c*x^2)^(2/3))/(7*2^
(1/3)*c*(-((c*(b*x + c*x^2))/b^2))^(2/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b
+ c*x))/b^2))^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*b^2*(b*x + c*x^2)^(2/3
)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b
 + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[
3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[
3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(14*2^(1/3)*c*(b + 2*c*x)*(-((c
*(b*x + c*x^2))/b^2))^(2/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1
/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)]) + (2^(1/6)
*3^(3/4)*b^2*(b*x + c*x^2)^(2/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3
))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b
+ c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))
^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))
/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/
(7*c*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(2/3)*Sqrt[-((1 - 2^(2/3)*(-((c
*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^
(1/3))^2)])
```

Rubi [A] time = 0.947094, antiderivative size = 781, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {622, 619, 195, 235, 304, 219, 1879}

$$\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}}-\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{7\sqrt[3]{2}c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}+\frac{\sqrt[6]{2}3^{3/4}b^2\left(bx+cx^2\right)^{2/3}\left(1-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(2/3),x]
```

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(2/3)*(b + 2*c*x)*(b*x + c*x^2)^(2/3))/(14*c*(-
((c*(b*x + c*x^2))/b^2))^(2/3)) - (3*(b + 2*c*x)*(b*x + c*x^2)^(2/3))/(7*2^
(1/3)*c*(-((c*(b*x + c*x^2))/b^2))^(2/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b
+ c*x))/b^2))^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*b^2*(b*x + c*x^2)^(2/3
)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b
 + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[
3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[
3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x
```

```
*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(14*2^(1/3)*c*(b + 2*c*x)*(-((c
*(b*x + c*x^2))/b^2))^(2/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1
/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)]) + (2^(1/6)
*3^(3/4)*b^2*(b*x + c*x^2)^(2/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3
))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b
+ c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))
^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))
/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/
(7*c*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(2/3)*Sqrt[-((1 - 2^(2/3)*(-((c
*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^
(1/3))^2)])
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
```

```
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(bx+cx^2\right)^{2/3} dx = \frac{\left(bx+cx^2\right)^{2/3} \int \left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{2/3} dx}{\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}}$$

$$= -\frac{\left(b^2\left(bx+cx^2\right)^{2/3}\right) \text{Subst}\left(\int \left(1-\frac{b^2x^2}{c^2}\right)^{2/3} dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{4\sqrt[3]{2}c^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}}$$

$$= \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}} - \frac{\left(b^2\left(bx+cx^2\right)^{2/3}\right) \text{Subst}\left(\int \frac{1}{\sqrt[3]{1-\frac{b^2x^2}{c^2}}} dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{7\sqrt[3]{2}c^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}}$$

$$= \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}} + \frac{\left(3\left(bx+cx^2\right)^{2/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right) \text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}} dx, x\right.}{14\sqrt[3]{2}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}}$$

$$= \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}} - \frac{\left(3\left(bx+cx^2\right)^{2/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right) \text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}} dx, x\right.}{14\sqrt[3]{2}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}}$$

$$= \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}} + \frac{3b^2\left(bx+cx^2\right)^{2/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\sqrt{-1-\frac{4cx(b+cx)}{b^2}}}{7\sqrt[3]{2}c(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)} -$$

Mathematica [C] time = 0.0094444, size = 45, normalized size = 0.06

$$\frac{3x(x(b+cx))^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{5}{3};\frac{8}{3};-\frac{cx}{b}\right)}{5\left(\frac{cx}{b}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(2/3),x]
```

```
[Out] (3*x*(x*(b + c*x))^(2/3)*Hypergeometric2F1[-2/3, 5/3, 8/3, -((c*x)/b)])/(5*
(1 + (c*x)/b)^(2/3))
```

Maple [F] time = 0.61, size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(2/3),x)
```

```
[Out] int((c*x^2+b*x)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx + cx^2\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(2/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(2/3), x)
```

3.37 $\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx$

Optimal. Leaf size=715

$$\frac{\sqrt[6]{2}3^{3/4}b^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right),4\sqrt{\cdots}\right.}{c(b+2cx)\sqrt[3]{bx+cx^2}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (-3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(1/3))/(2^(1/3)*c*(b*x + c*x^2)^
(1/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))) - (3*3^(1/4)*
Sqrt[2 + Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(1/3)*(1 - 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) +
 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(
b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(
b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3
))], -7 + 4*Sqrt[3]])/(2*2^(1/3)*c*(b + 2*c*x)*(b*x + c*x^2)^(1/3)*Sqrt[-((
1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*
(b + c*x))/b^2))^(1/3))^2)]) + (2^(1/6)*3^(3/4)*b^2*(-((c*(b*x + c*x^2))/b^
2))^(1/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1
 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)
*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c
*x^2)^(1/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3
] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 0.86049, antiderivative size = 715, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {622, 619, 235, 304, 219, 1879}

$$-\frac{3(b+2cx)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}{\sqrt[3]{2}c\sqrt[3]{bx+cx^2}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}+\frac{\sqrt[6]{2}3^{3/4}b^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b\cdots}{b^{\cdots}}}\cdots}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)\cdots}}\cdots}{c(b+2cx)\sqrt[3]{bx+cx^2}\sqrt{-\frac{1-2^{2/3}\cdots}{\left(-2^{2/3}\sqrt[3]{-\cdots}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-1/3),x]
```


```
[Out] (-3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(1/3))/(2^(1/3)*c*(b*x + c*x^2)^
(1/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))) - (3*3^(1/4)*
Sqrt[2 + Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(1/3)*(1 - 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) +
 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(
b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(
b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3
))], -7 + 4*Sqrt[3]])/(2*2^(1/3)*c*(b + 2*c*x)*(b*x + c*x^2)^(1/3)*Sqrt[-((
1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*
(b + c*x))/b^2))^(1/3))^2)]) + (2^(1/6)*3^(3/4)*b^2*(-((c*(b*x + c*x^2))/b^
```

```
2))^(1/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1
 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)
*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c
*x^2)^(1/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3
] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx = \frac{\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[3]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}}\,dx}{\sqrt[3]{bx+cx^2}}$$

$$= -\frac{\left(b^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-\frac{b^2x^2}{c^2}}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{\sqrt[3]{2}c^2\sqrt[3]{bx+cx^2}}$$

$$= \frac{\left(3\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, 2^{2/3}\sqrt[3]{-\frac{cx\left(1+\frac{cx}{b}\right)}{b}}\right)}{2\sqrt[3]{2}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)\sqrt[3]{bx+cx^2}}$$

$$= -\frac{\left(3\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, 2^{2/3}\sqrt[3]{-\frac{cx\left(1+\frac{cx}{b}\right)}{b}}\right)}{2\sqrt[3]{2}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)\sqrt[3]{bx+cx^2}} + \frac{\left(3\sqrt{2+\sqrt{3}}\sqrt[3]{-\frac{c(bx\cdots}{b\cdots}}\right.}{\cdots}$$

$$= \frac{3b^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\sqrt{-1-\frac{4cx(b+cx)}{b^2}}}{\sqrt[3]{2}c(b+2cx)\sqrt[3]{bx+cx^2}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)} - \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2\cdots}{b^2}}\cdots}{2\sqrt[3]{2}c\cdots}$$

Mathematica [C] time = 0.0102843, size = 45, normalized size = 0.06

$$\frac{3x\sqrt[3]{\frac{cx}{b}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{cx}{b}\right)}{2\sqrt[3]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-1/3),x]
```

```
[Out] (3*x*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((c*x)/b)])/(2*(
x*(b + c*x))^(1/3))
```

Maple [F] time = 0.699, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(1/3),x)
```

```
[Out] int(1/(c*x^2+b*x)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(cx^2+bx\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(-1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(1/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-1/3), x)
```

3.38 $\int \frac{1}{(bx+cx^2)^{4/3}}\,dx$

Optimal. Leaf size=773

$$-\frac{2\sqrt[6]{2}\,3^{3/4}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\Bigg(\sin^{-1}\Bigg(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}}{c(b+2cx)\left(bx+cx^2\right)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(4/3))/(c*(-((c*x*(b + c*x))/b^2)
)^(1/3)*(b*x + c*x^2)^(4/3)) + (3*2^(2/3)*(b + 2*c*x)*(-((c*(b*x + c*x^2))/
b^2))^(4/3))/(c*(b*x + c*x^2)^(4/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))) + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^
2))^(4/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1
 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)
*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(2^(1/3)*c*(b + 2*c*x)*
(b*x + c*x^2)^(4/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1
- Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)]) - (2*2^(1/6)*3^(3/
4)*b^2*(-((c*(b*x + c*x^2))/b^2))^(4/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2
))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((
c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))
^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))
^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqr
t[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)^(4/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b +
c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2
)])
```

Rubi [A] time = 0.946652, antiderivative size = 773, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {622, 619, 199, 235, 304, 219, 1879}

$$\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(bx+cx^2\right)^{4/3}}+\frac{3\,2^{2/3}(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\left(bx+cx^2\right)^{4/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}-\frac{2\sqrt[6]{2}\,3^{3/4}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\Bigg(1-2^{2/3}\sqrt[3]{-\frac{cx(b+c}{b^2}}}{c(b+2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-4/3),x]
```

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(4/3))/(c*(-((c*x*(b + c*x))/b^2)
)^(1/3)*(b*x + c*x^2)^(4/3)) + (3*2^(2/3)*(b + 2*c*x)*(-((c*(b*x + c*x^2))/
b^2))^(4/3))/(c*(b*x + c*x^2)^(4/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))) + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^
2))^(4/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1
```

```
 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)
*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(2^(1/3)*c*(b + 2*c*x)*
(b*x + c*x^2)^(4/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1
- Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)]) - (2*2^(1/6)*3^(3/
4)*b^2*(-((c*(b*x + c*x^2))/b^2))^(4/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2
))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((
c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))
^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))
^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqr
t[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)^(4/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b +
c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2
)])
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
```

```
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(bx+cx^2\right)^{4/3}}\,dx = \frac{\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{4/3}}\,dx}{\left(bx+cx^2\right)^{4/3}}$$

$$= -\frac{\left(2\,2^{2/3}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{4/3}}\,dx,x,-\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\left(bx+cx^2\right)^{4/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(bx+cx^2\right)^{4/3}} + \frac{\left(2^{2/3}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-\frac{b^2x^2}{c^2}}}\,dx,x,-\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\left(bx+cx^2\right)^{4/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(bx+cx^2\right)^{4/3}} - \frac{\left(3\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx,x,2^{2/3}\sqrt[3]{-\cdots}\right.}{\sqrt[3]{2}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)\left(bx+cx^2\right)^{4/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(bx+cx^2\right)^{4/3}} + \frac{\left(3\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx,x,2^{2/3}\sqrt[3]{\cdots}\right.}{\sqrt[3]{2}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)\left(bx+cx^2\right)^{4/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(bx+cx^2\right)^{4/3}} - \frac{3\,2^{2/3}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{4/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\sqrt{-1-\frac{4cx(b+cx)}{b^2}}}{c(b+2cx)\left(bx+cx^2\right)^{4/3}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)} + \frac{3\sqrt[4]{3}\sqrt{\cdots}}{\cdots}$$

Mathematica [C] time = 0.011627, size = 45, normalized size = 0.06

$$-\frac{3\sqrt[3]{\frac{cx}{b}+1}\,{}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{2}{3};-\frac{cx}{b}\right)}{b\sqrt[3]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-4/3),x]
```

```
[Out] (-3*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 2/3, -((c*x)/b)])/(b*(
x*(b + c*x))^(1/3))
```

Maple [F] time = 0.928, size = 0, normalized size = 0.

$$\int \left(c x^2 + b x\right)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(4/3),x)
```

```
[Out] int(1/(c*x^2+b*x)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c x^2 + b x\right)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c x^2 + b x\right)^{\frac{2}{3}}}{c^2 x^4 + 2 \, b c x^3 + b^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(2/3)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x + c x^2\right)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(4/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c x^2 + b x\right)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-4/3), x)
```

3.39 $\int \frac{1}{\left(bx+cx^2\right)^{7/3}}\,dx$

Optimal. Leaf size=838

$$\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)\left(-\frac{c(cx^2}{b}\right.}{2\sqrt[3]{2}c(b+2cx)\left(cx^2+bx\right)^{7/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(7/3))/(4*c*(-((c*x*(b + c*x))/b^
2))^(4/3)*(b*x + c*x^2)^(7/3)) + (15*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))
^(7/3))/(2*c*(-((c*x*(b + c*x))/b^2))^(1/3)*(b*x + c*x^2)^(7/3)) + (15*(b +
 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(7/3))/(2^(1/3)*c*(b*x + c*x^2)^(7/3)*(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))) + (15*3^(1/4)*Sqrt[2
+ Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(7/3)*(1 - 2^(2/3)*(-((c*x*(b + c
*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1
/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7
 + 4*Sqrt[3]])/(2*2^(1/3)*c*(b + 2*c*x)*(b*x + c*x^2)^(7/3)*Sqrt[-((1 - 2^(
2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*
x))/b^2))^(1/3))^2)]) - (5*2^(1/6)*3^(3/4)*b^2*(-((c*(b*x + c*x^2))/b^2))^(
7/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sq
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sq
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)
^(7/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2
^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 1.03573, antiderivative size = 838, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {622, 619, 199, 235, 304, 219, 1879}

$$\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)\left(-\frac{c(cx^2}{b}\right.}{2\sqrt[3]{2}c(b+2cx)\left(cx^2+bx\right)^{7/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-7/3),x]
```

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(7/3))/(4*c*(-((c*x*(b + c*x))/b^
2))^(4/3)*(b*x + c*x^2)^(7/3)) + (15*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))
^(7/3))/(2*c*(-((c*x*(b + c*x))/b^2))^(1/3)*(b*x + c*x^2)^(7/3)) + (15*(b +
 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(7/3))/(2^(1/3)*c*(b*x + c*x^2)^(7/3)*(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))) + (15*3^(1/4)*Sqrt[2
+ Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(7/3)*(1 - 2^(2/3)*(-((c*x*(b + c
```

```
*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1
/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7
 + 4*Sqrt[3]])/(2*2^(1/3)*c*(b + 2*c*x)*(b*x + c*x^2)^(7/3)*Sqrt[-((1 - 2^(
2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*
x))/b^2))^(1/3))^2)]) - (5*2^(1/6)*3^(3/4)*b^2*(-((c*(b*x + c*x^2))/b^2))^(
7/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sq
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sq
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)
^(7/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2
^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(bx+cx^2\right)^{7/3}}\,dx = \frac{\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{7/3}}\,dx}{\left(bx+cx^2\right)^{7/3}}$$

$$= -\frac{\left(8\ 2^{2/3}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{7/3}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\left(bx+cx^2\right)^{7/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}\left(bx+cx^2\right)^{7/3}} - \frac{\left(5\ 2^{2/3}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{4/3}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\left(bx+cx^2\right)^{7/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}\left(bx+cx^2\right)^{7/3}} + \frac{15(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}}{2c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(bx+cx^2\right)^{7/3}} + \frac{\left(5b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-\ldots}}\ldots\right.}{\sqrt[3]{2}c^2\left(bx+cx^2\right)^{7/\ldots}}$$

$$= \frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}\left(bx+cx^2\right)^{7/3}} + \frac{15(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}}{2c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(bx+cx^2\right)^{7/3}} - \frac{\left(15\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\ldots\right.}{2\sqrt[3]{2}\left(-\frac{c}{b}-\ldots\right.}$$

$$= \frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}\left(bx+cx^2\right)^{7/3}} + \frac{15(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}}{2c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(bx+cx^2\right)^{7/3}} + \frac{\left(15\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\ldots\right.}{2\sqrt[3]{2}\left(-\frac{c}{b}-\ldots\right.}$$

$$= \frac{3(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}\left(bx+cx^2\right)^{7/3}} + \frac{15(b+2cx)\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}}{2c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(bx+cx^2\right)^{7/3}} - \frac{15b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{7/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x\ldots}{b^2}}}{\sqrt[3]{2}c(b+2cx)\left(bx+cx^2\right)^{7/3}\left(1-\sqrt{3}\ldots\right.}$$

Mathematica [C] time = 0.0121808, size = 50, normalized size = 0.06

$$-\frac{3\sqrt[3]{\frac{cx}{b}+1}\,{}_2F_1\left(-\frac{4}{3},\frac{7}{3};-\frac{1}{3};-\frac{cx}{b}\right)}{4b^2x\sqrt[3]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-7/3),x]
```

```
[Out] (-3*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[-4/3, 7/3, -1/3, -((c*x)/b)])/(4*
b^2*x*(x*(b + c*x))^(1/3))
```

Maple [F] time = 1.415, size = 0, normalized size = 0.

$$\int \left(c x^{2}+b x\right)^{-\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(7/3),x)
```

```
[Out] int(1/(c*x^2+b*x)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c x^{2}+b x\right)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-7/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c x^{2}+b x\right)^{\frac{2}{3}}}{c^{3} x^{6}+3\, b c^{2} x^{5}+3\, b^{2} c x^{4}+b^{3} x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(2/3)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x+c x^{2}\right)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(7/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-7/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-7/3), x)
```

3.40 $\int \left(bx+cx^2\right)^{5/4} dx$

Optimal. Leaf size=119

$$\frac{5b^5\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right),2\right)}{84\sqrt{2}c^3\left(bx+cx^2\right)^{3/4}}-\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2}+\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}$$

```
[Out] (-5*b^2*(b + 2*c*x)*(b*x + c*x^2)^(1/4))/(84*c^2) + ((b + 2*c*x)*(b*x + c*x
^2)^(5/4))/(7*c) + (5*b^5*(-((c*(b*x + c*x^2))/b^2))^(3/4)*EllipticF[ArcSin
[1 + (2*c*x)/b]/2, 2])/(84*Sqrt[2]*c^3*(b*x + c*x^2)^(3/4))
```

Rubi [A] time = 0.0462408, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {612, 622, 619, 232}

$$-\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2}+\frac{5b^5\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{84\sqrt{2}c^3\left(bx+cx^2\right)^{3/4}}+\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/4),x]
```



```
[Out] (-5*b^2*(b + 2*c*x)*(b*x + c*x^2)^(1/4))/(84*c^2) + ((b + 2*c*x)*(b*x + c*x
^2)^(5/4))/(7*c) + (5*b^5*(-((c*(b*x + c*x^2))/b^2))^(3/4)*EllipticF[ArcSin
[1 + (2*c*x)/b]/2, 2])/(84*Sqrt[2]*c^3*(b*x + c*x^2)^(3/4))
```


Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \left(bx+cx^2\right)^{5/4} dx = \frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c} - \frac{\left(5b^2\right)\int \sqrt[4]{bx+cx^2}\,dx}{28c}$$
$$= -\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c} + \frac{\left(5b^4\right)\int \frac{1}{\left(bx+cx^2\right)^{3/4}}\,dx}{336c^2}$$
$$= -\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c} + \frac{\left(5b^4\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}\right)\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{3/4}}\,dx}{336c^2\left(bx+cx^2\right)^{3/4}}$$
$$= -\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c} - \frac{\left(5b^6\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{3/4}}\,dx, x}{168\sqrt{2}c^4\left(bx+cx^2\right)^{3/4}}$$
$$= -\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c} + \frac{5b^5\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{84\sqrt{2}c^3\left(bx+cx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0149402, size = 48, normalized size = 0.4

$$\frac{4bx^2\sqrt[4]{x(b+cx)}\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4};\frac{13}{4};-\frac{cx}{b}\right)}{9\sqrt[4]{\frac{cx}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/4),x]
```

```
[Out] (4*b*x^2*(x*(b + c*x))^(1/4)*Hypergeometric2F1[-5/4, 9/4, 13/4, -((c*x)/b)]
)/(9*(1 + (c*x)/b)^(1/4))
```

Maple [F] time = 0.477, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/4),x)
```

```
[Out] int((c*x^2+b*x)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/4),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^2+bx\right)^{\frac{5}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/4),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(5/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx+cx^2\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/4),x)

[Out] Integral((b*x + c*x**2)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/4),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(5/4), x)
```

3.41 $\int \left(bx+cx^2\right)^{3/4}\,dx$

Optimal. Leaf size=90

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{3b^3\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{10\sqrt{2}c^2\sqrt[4]{bx+cx^2}}$$

```
[Out] ((b + 2*c*x)*(b*x + c*x^2)^(3/4))/(5*c) - (3*b^3*(-((c*(b*x + c*x^2))/b^2))
^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(10*Sqrt[2]*c^2*(b*x + c*x^2)
^(1/4))
```

Rubi [A] time = 0.0339669, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {612, 622, 619, 228}

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{3b^3\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{10\sqrt{2}c^2\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/4),x]
```

```
[Out] ((b + 2*c*x)*(b*x + c*x^2)^(3/4))/(5*c) - (3*b^3*(-((c*(b*x + c*x^2))/b^2))
^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(10*Sqrt[2]*c^2*(b*x + c*x^2)
^(1/4))
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \left(bx+cx^2\right)^{3/4} dx = \frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c} - \frac{\left(3b^2\right)\int \frac{1}{\sqrt[4]{bx+cx^2}}dx}{20c}$$

$$= \frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c} - \frac{\left(3b^2\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right)\int \frac{1}{\sqrt[4]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}}dx}{20c\sqrt[4]{bx+cx^2}}$$

$$= \frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c} + \frac{\left(3b^4\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{b^2x^2}{c^2}}}dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2}\right)}{20\sqrt{2}c^3\sqrt[4]{bx+cx^2}}$$

$$= \frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c} - \frac{3b^3\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{10\sqrt{2}c^2\sqrt[4]{bx+cx^2}}$$

Mathematica [C] time = 0.0127553, size = 45, normalized size = 0.5

$$\frac{4x(x(b+cx))^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4};\frac{11}{4};-\frac{cx}{b}\right)}{7\left(\frac{cx}{b}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/4),x]
```

```
[Out] (4*x*(x*(b + c*x))^(3/4)*Hypergeometric2F1[-3/4, 7/4, 11/4, -((c*x)/b)])/(7
*(1 + (c*x)/b)^(3/4))
```

Maple [F] time = 0.656, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/4),x)
```

```
[Out] int((c*x^2+b*x)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx\right)^{\frac{3}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx+cx^2\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(3/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(3/4), x)
```

3.42 $\int \sqrt[4]{bx + cx^2}\, dx$

Optimal. Leaf size=90

$$\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c} - \frac{b^3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right),2\right)}{3\sqrt{2}c^2\left(bx+cx^2\right)^{3/4}}$$

```
[Out] ((b + 2*c*x)*(b*x + c*x^2)^(1/4))/(3*c) - (b^3*(-((c*(b*x + c*x^2))/b^2))^(
3/4)*EllipticF[ArcSin[1 + (2*c*x)/b]/2, 2])/(3*Sqrt[2]*c^2*(b*x + c*x^2)^(3
/4))
```

Rubi [A] time = 0.0327646, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {612, 622, 619, 232}

$$\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c} - \frac{b^3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{3\sqrt{2}c^2\left(bx+cx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(1/4),x]
```

```
[Out] ((b + 2*c*x)*(b*x + c*x^2)^(1/4))/(3*c) - (b^3*(-((c*(b*x + c*x^2))/b^2))^(
3/4)*EllipticF[ArcSin[1 + (2*c*x)/b]/2, 2])/(3*Sqrt[2]*c^2*(b*x + c*x^2)^(3
/4))
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \sqrt[4]{bx+cx^2}\,dx = \frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c} - \frac{b^2\int \frac{1}{(bx+cx^2)^{3/4}}\,dx}{12c}$$

$$= \frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c} - \frac{\left(b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\right)\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{3/4}}\,dx}{12c\,(bx+cx^2)^{3/4}}$$

$$= \frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c} + \frac{\left(b^4\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{3/4}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{6\sqrt{2}c^3\,(bx+cx^2)^{3/4}}$$

$$= \frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c} - \frac{b^3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{3\sqrt{2}c^2\,(bx+cx^2)^{3/4}}$$

Mathematica [C] time = 0.009837, size = 45, normalized size = 0.5

$$\frac{4x\sqrt[4]{x(b+cx)}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{9}{4};-\frac{cx}{b}\right)}{5\sqrt[4]{\frac{cx}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(1/4),x]
```

```
[Out] (4*x*(x*(b + c*x))^(1/4)*Hypergeometric2F1[-1/4, 5/4, 9/4, -((c*x)/b)])/(5*
(1 + (c*x)/b)^(1/4))
```

Maple [F] time = 0.401, size = 0, normalized size = 0.

$$\int \sqrt[4]{cx^2+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/4),x)
```

```
[Out] int((c*x^2+b*x)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^2+bx)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[4]{bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{1}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(1/4), x)
```

3.43 $\int \frac{1}{\sqrt[4]{bx+cx^2}} dx$

Optimal. Leaf size=58

$$\frac{\sqrt{2}b\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{c\sqrt[4]{bx+cx^2}}$$

```
[Out] (Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]
/2, 2])/(c*(b*x + c*x^2)^(1/4))
```

Rubi [A] time = 0.0226108, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {622, 619, 228}

$$\frac{\sqrt{2}b\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{c\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-1/4),x]
```

```
[Out] (Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]
/2, 2])/(c*(b*x + c*x^2)^(1/4))
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx = \frac{\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}}\,dx}{\sqrt[4]{bx+cx^2}}$$

$$= -\frac{\left(b^2\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{b^2x^2}{c^2}}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{\sqrt{2}c^2\sqrt[4]{bx+cx^2}}$$

$$= \frac{\sqrt{2}b\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{c\sqrt[4]{bx+cx^2}}$$

Mathematica [C] time = 0.0098401, size = 45, normalized size = 0.78

$$\frac{4x\sqrt[4]{\frac{cx}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{cx}{b}\right)}{3\sqrt[4]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-1/4),x]
```

```
[Out] (4*x*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((c*x)/b)])/(3*(
x*(b + c*x))^(1/4))
```

Maple [F] time = 0.608, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(1/4),x)
```

```
[Out] int(1/(c*x^2+b*x)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+bx)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(cx^2+bx\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(-1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(1/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(-1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-1/4), x)
```

3.44 $\int \frac{1}{\left(bx+cx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=59

$$\frac{2\sqrt{2}b\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right),2\right)}{c\left(bx+cx^2\right)^{3/4}}$$

```
[Out] (2*Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(3/4)*EllipticF[ArcSin[1 + (2*c*x)/
b]/2, 2])/(c*(b*x + c*x^2)^(3/4))
```

Rubi [A] time = 0.0234089, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {622, 619, 232}

$$\frac{2\sqrt{2}b\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{c\left(bx+cx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-3/4),x]
```

```
[Out] (2*Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(3/4)*EllipticF[ArcSin[1 + (2*c*x)/
b]/2, 2])/(c*(b*x + c*x^2)^(3/4))
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(bx+cx^2\right)^{3/4}}\,dx = \frac{\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{3/4}}\,dx}{\left(bx+cx^2\right)^{3/4}}$$

$$= -\frac{\left(\sqrt{2}b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{3/4}}\,dx,x,-\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\left(bx+cx^2\right)^{3/4}}$$

$$= \frac{2\sqrt{2}b\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{c\left(bx+cx^2\right)^{3/4}}$$

Mathematica [C] time = 0.0102803, size = 43, normalized size = 0.73

$$\frac{4x\left(\frac{cx}{b}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{cx}{b}\right)}{(x(b+cx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-3/4),x]
```

```
[Out] (4*x*(1 + (c*x)/b)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((c*x)/b)])/(x*(
b + c*x))^(3/4)
```

Maple [F] time = 0.678, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(3/4),x)
```

```
[Out] int(1/(c*x^2+b*x)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(cx^2+bx\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(-3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx+cx^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(3/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(-3/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-3/4), x)
```

3.45 $\int \frac{1}{\left(bx+cx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=83

$$\frac{4\sqrt{2}\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

```
[Out] (-4*(b + 2*c*x))/(b^2*(b*x + c*x^2)^(1/4)) + (4*Sqrt[2]*(-((c*(b*x + c*x^2)
)/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(b*(b*x + c*x^2)^(1/4)
)
```

Rubi [A] time = 0.0316459, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {614, 622, 619, 228}

$$\frac{4\sqrt{2}\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-5/4),x]
```

```
[Out] (-4*(b + 2*c*x))/(b^2*(b*x + c*x^2)^(1/4)) + (4*Sqrt[2]*(-((c*(b*x + c*x^2)
)/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(b*(b*x + c*x^2)^(1/4)
)
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(bx+cx^2\right)^{5/4}}\,dx = -\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}+\frac{(4c)\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx}{b^2}$$

$$= -\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}+\frac{\left(4c\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\right)\int \frac{1}{\sqrt[4]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}}\,dx}{b^2\sqrt[4]{bx+cx^2}}$$

$$= -\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}-\frac{\left(2\sqrt{2}\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{b^2x^2}{c^2}}}\,dx,x,-\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c\sqrt[4]{bx+cx^2}}$$

$$= -\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}+\frac{4\sqrt{2}\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}$$

Mathematica [C] time = 0.0129641, size = 45, normalized size = 0.54

$$-\frac{4\sqrt[4]{\frac{cx}{b}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};-\frac{cx}{b}\right)}{b\sqrt[4]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-5/4),x]
```

```
[Out] (-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, -((c*x)/b)])/(b*(
x*(b + c*x))^(1/4))
```

Maple [F] time = 0.861, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(5/4),x)
```

```
[Out] int(1/(c*x^2+b*x)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx\right)^{\frac{3}{4}}}{c^2x^4+2\,bcx^3+b^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/4)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx+cx^2\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(5/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(-5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-5/4), x)
```

3.46 $\int \frac{1}{(bx+cx^2)^{9/4}}\,dx$

Optimal. Leaf size=115

$$\frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/4}}-\frac{48\sqrt{2}c\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{5b^3\sqrt[4]{bx+cx^2}}$$

```
[Out] (-4*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/4)) + (48*c*(b + 2*c*x))/(5*b^4*(b
*x + c*x^2)^(1/4)) - (48*Sqrt[2]*c*(-((c*(b*x + c*x^2))/b^2))^(1/4)*Ellipti
cE[ArcSin[1 + (2*c*x)/b]/2, 2])/(5*b^3*(b*x + c*x^2)^(1/4))
```

Rubi [A] time = 0.0435125, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {614, 622, 619, 228}

$$\frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/4}}-\frac{48\sqrt{2}c\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{5b^3\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-9/4),x]
```

```
[Out] (-4*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/4)) + (48*c*(b + 2*c*x))/(5*b^4*(b
*x + c*x^2)^(1/4)) - (48*Sqrt[2]*c*(-((c*(b*x + c*x^2))/b^2))^(1/4)*Ellipti
cE[ArcSin[1 + (2*c*x)/b]/2, 2])/(5*b^3*(b*x + c*x^2)^(1/4))
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(bx+cx^2\right)^{9/4}}\,dx = -\frac{4(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/4}} - \frac{(12c)\int \frac{1}{\left(bx+cx^2\right)^{5/4}}\,dx}{5b^2}$$

$$= -\frac{4(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/4}} + \frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}} - \frac{\left(48c^2\right)\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx}{5b^4}$$

$$= -\frac{4(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/4}} + \frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}} - \frac{\left(48c^2\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\right)\int \frac{1}{\sqrt[4]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}}\,dx}{5b^4\sqrt[4]{bx+cx^2}}$$

$$= -\frac{4(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/4}} + \frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}} + \frac{\left(24\sqrt{2}\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{b^2x^2}{c^2}}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{5b^2\sqrt[4]{bx+cx^2}}$$

$$= -\frac{4(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/4}} + \frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}} - \frac{48\sqrt{2}c\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{5b^3\sqrt[4]{bx+cx^2}}$$

Mathematica [C] time = 0.0129909, size = 50, normalized size = 0.43

$$-\frac{4\sqrt[4]{\frac{cx}{b}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4};-\frac{1}{4};-\frac{cx}{b}\right)}{5b^2x\sqrt[4]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-9/4),x]
```

```
[Out] (-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-5/4, 9/4, -1/4, -((c*x)/b)])/(5*
b^2*x*(x*(b + c*x))^(1/4))
```

Maple [F] time = 1.42, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(9/4),x)
```

```
[Out] int(1/(c*x^2+b*x)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cx^2+bx\right)^{\frac{3}{4}}}{c^3x^6+3\,bc^2x^5+3\,b^2cx^4+b^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/4)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx+cx^2\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(9/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(-9/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-9/4), x)
```

3.47 $\int \frac{1}{\left(bx+cx^2\right)^{13/4}}\,dx$

Optimal. Leaf size=146

$$-\frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}}+\frac{448\sqrt{2}c^2\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{15b^5\sqrt[4]{bx+cx^2}}+\frac{112c(b+2cx)}{45b^4\left(bx+cx^2\right)^{5/4}}-\frac{4(b+2cx)}{9b^2\left(bx+cx^2\right)^{9/4}}$$

```
[Out] (-4*(b + 2*c*x))/(9*b^2*(b*x + c*x^2)^(9/4)) + (112*c*(b + 2*c*x))/(45*b^4*
(b*x + c*x^2)^(5/4)) - (448*c^2*(b + 2*c*x))/(15*b^6*(b*x + c*x^2)^(1/4)) +
 (448*Sqrt[2]*c^2*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*
c*x)/b]/2, 2])/(15*b^5*(b*x + c*x^2)^(1/4))
```

Rubi [A] time = 0.0601218, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {614, 622, 619, 228}

$$-\frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}}+\frac{448\sqrt{2}c^2\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{15b^5\sqrt[4]{bx+cx^2}}+\frac{112c(b+2cx)}{45b^4\left(bx+cx^2\right)^{5/4}}-\frac{4(b+2cx)}{9b^2\left(bx+cx^2\right)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-13/4),x]
```

```
[Out] (-4*(b + 2*c*x))/(9*b^2*(b*x + c*x^2)^(9/4)) + (112*c*(b + 2*c*x))/(45*b^4*
(b*x + c*x^2)^(5/4)) - (448*c^2*(b + 2*c*x))/(15*b^6*(b*x + c*x^2)^(1/4)) +
 (448*Sqrt[2]*c^2*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*
c*x)/b]/2, 2])/(15*b^5*(b*x + c*x^2)^(1/4))
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
```

```
GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(bx+cx^2\right)^{13/4}}\,dx = -\frac{4(b+2cx)}{9b^2\left(bx+cx^2\right)^{9/4}} - \frac{(28c)\int \frac{1}{\left(bx+cx^2\right)^{9/4}}\,dx}{9b^2}$$

$$= -\frac{4(b+2cx)}{9b^2\left(bx+cx^2\right)^{9/4}} + \frac{112c(b+2cx)}{45b^4\left(bx+cx^2\right)^{5/4}} + \frac{\left(112c^2\right)\int \frac{1}{\left(bx+cx^2\right)^{5/4}}\,dx}{15b^4}$$

$$= -\frac{4(b+2cx)}{9b^2\left(bx+cx^2\right)^{9/4}} + \frac{112c(b+2cx)}{45b^4\left(bx+cx^2\right)^{5/4}} - \frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}} + \frac{\left(448c^3\right)\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx}{15b^6}$$

$$= -\frac{4(b+2cx)}{9b^2\left(bx+cx^2\right)^{9/4}} + \frac{112c(b+2cx)}{45b^4\left(bx+cx^2\right)^{5/4}} - \frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}} + \frac{\left(448c^3\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\right)\int \frac{1}{\sqrt[4]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}}$$

$$= -\frac{4(b+2cx)}{9b^2\left(bx+cx^2\right)^{9/4}} + \frac{112c(b+2cx)}{45b^4\left(bx+cx^2\right)^{5/4}} - \frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}} - \frac{\left(224\sqrt{2}c\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\right)\operatorname{Subst}\left(\int\right.}{15b^4\sqrt[4]{bx+c}}$$

$$= -\frac{4(b+2cx)}{9b^2\left(bx+cx^2\right)^{9/4}} + \frac{112c(b+2cx)}{45b^4\left(bx+cx^2\right)^{5/4}} - \frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}} + \frac{448\sqrt{2}c^2\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\right.\right.}{15b^5\sqrt[4]{bx+cx^2}}$$

Mathematica [C] time = 0.0119771, size = 50, normalized size = 0.34

$$-\frac{4\sqrt[4]{\frac{cx}{b}+1}\,{}_2F_1\left(-\frac{9}{4},\frac{13}{4};-\frac{5}{4};-\frac{cx}{b}\right)}{9b^3x^2\sqrt[4]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-13/4),x]
```

```
[Out] (-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-9/4, 13/4, -5/4, -((c*x)/b)])/(9
*b^3*x^2*(x*(b + c*x))^(1/4))
```

Maple [F] time = 0.652, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{-\frac{13}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(13/4),x)
```

```
[Out] int(1/(c*x^2+b*x)^(13/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(13/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-13/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx\right)^{\frac{3}{4}}}{c^4x^8+4\,bc^3x^7+6\,b^2c^2x^6+4\,b^3cx^5+b^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(13/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/4)/(c^4*x^8 + 4*b*c^3*x^7 + 6*b^2*c^2*x^6 + 4*b^3
*c*x^5 + b^4*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx+cx^2\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(13/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(-13/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(13/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-13/4), x)
```

3.48 $\int \left(bx+cx^2\right)^p\, dx$

Optimal. Leaf size=55

$$-\frac{\left(-\frac{cx}{b}\right)^{-p-1}\left(bx+cx^2\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{b+cx}{b}\right)}{b(p+1)}$$

```
[Out] -(((-((c*x)/b))^(-1 - p)*(b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p,
 2 + p, (b + c*x)/b])/(b*(1 + p)))
```

Rubi [A] time = 0.0104559, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {624}

$$-\frac{\left(-\frac{cx}{b}\right)^{-p-1}\left(bx+cx^2\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{b+cx}{b}\right)}{b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^p,x]
```

```
[Out] -(((-((c*x)/b))^(-1 - p)*(b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p,
 2 + p, (b + c*x)/b])/(b*(1 + p)))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int \left(bx+cx^2\right)^p\, dx=-\frac{\left(-\frac{cx}{b}\right)^{-1-p}\left(bx+cx^2\right)^{1+p}{}_2F_1\left(-p,1+p;2+p;\frac{b+cx}{b}\right)}{b(1+p)}$$

Mathematica [A] time = 0.0099882, size = 45, normalized size = 0.82

$$\frac{x(x(b+cx))^p\left(\frac{cx}{b}+1\right)^{-p}{}_2F_1\left(-p,p+1;p+2;-\frac{cx}{b}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^p,x]
```

```
[Out] (x*(x*(b + c*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x)/b)])/((1 + p
)*(1 + (c*x)/b)^p)
```

Maple [F] time = 0.441, size = 0, normalized size = 0.

$$\int \left(c x^2 + b x\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^p,x)
```

```
[Out] int((c*x^2+b*x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^2 + b x\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c x^2 + b x\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x + c x^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**p,x)
```

```
[Out] Integral((b*x + c*x**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^2 + b x\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^p, x)
```

3.49 $\int \left(a+cx^2\right)^4 dx$

Optimal. Leaf size=51

$$\frac{6}{5}a^2c^2x^5+\frac{4}{3}a^3cx^3+a^4x+\frac{4}{7}ac^3x^7+\frac{c^4x^9}{9}$$

```
[Out] a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (c^4*x^9)/9
```

Rubi [A] time = 0.0173736, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$\frac{6}{5}a^2c^2x^5+\frac{4}{3}a^3cx^3+a^4x+\frac{4}{7}ac^3x^7+\frac{c^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^4,x]
```

```
[Out] a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (c^4*x^9)/9
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)^4 dx &= \int \left(a^4+4a^3cx^2+6a^2c^2x^4+4ac^3x^6+c^4x^8\right) dx\\ &= a^4x+\frac{4}{3}a^3cx^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}ac^3x^7+\frac{c^4x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0017068, size = 51, normalized size = 1.

$$\frac{6}{5}a^2c^2x^5+\frac{4}{3}a^3cx^3+a^4x+\frac{4}{7}ac^3x^7+\frac{c^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^4,x]
```

```
[Out] a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (c^4*x^9)/9
```

Maple [A] time = 0.045, size = 44, normalized size = 0.9

$$a^4x+\frac{4\,a^3cx^3}{3}+\frac{6\,a^2c^2x^5}{5}+\frac{4\,ac^3x^7}{7}+\frac{c^4x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^4,x)
```

```
[Out] a^4*x+4/3*a^3*c*x^3+6/5*a^2*c^2*x^5+4/7*a*c^3*x^7+1/9*c^4*x^9
```

Maxima [A] time = 1.16844, size = 58, normalized size = 1.14

$$\frac{1}{9}c^4x^9 + \frac{4}{7}ac^3x^7 + \frac{6}{5}a^2c^2x^5 + \frac{4}{3}a^3cx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a^4*x
```

Fricas [A] time = 1.79691, size = 96, normalized size = 1.88

$$\frac{1}{9}x^9c^4 + \frac{4}{7}x^7c^3a + \frac{6}{5}x^5c^2a^2 + \frac{4}{3}x^3ca^3 + xa^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^4 + 4/7*x^7*c^3*a + 6/5*x^5*c^2*a^2 + 4/3*x^3*c*a^3 + x*a^4
```

Sympy [A] time = 0.074117, size = 49, normalized size = 0.96

$$a^4x + \frac{4a^3cx^3}{3} + \frac{6a^2c^2x^5}{5} + \frac{4ac^3x^7}{7} + \frac{c^4x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**4,x)
```

```
[Out] a**4*x + 4*a**3*c*x**3/3 + 6*a**2*c**2*x**5/5 + 4*a*c**3*x**7/7 + c**4*x**9
/9
```

Giac [A] time = 1.23238, size = 58, normalized size = 1.14

$$\frac{1}{9}c^4x^9 + \frac{4}{7}ac^3x^7 + \frac{6}{5}a^2c^2x^5 + \frac{4}{3}a^3cx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a^4*x
```

3.50 $\int \left(a+cx^2\right)^3 \, dx$

Optimal. Leaf size=35

$$a^2cx^3 + a^3x + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7}$$

```
[Out] a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^7)/7
```

Rubi [A] time = 0.0114554, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2cx^3 + a^3x + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3,x]
```

```
[Out] a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^7)/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)^3 \, dx &= \int \left(a^3+3a^2cx^2+3ac^2x^4+c^3x^6\right) dx\\ &= a^3x + a^2cx^3 + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0012346, size = 35, normalized size = 1.

$$a^2cx^3 + a^3x + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3,x]
```

```
[Out] a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^7)/7
```

Maple [A] time = 0.046, size = 32, normalized size = 0.9

$$xa^3 + a^2cx^3 + \frac{3\,ac^2x^5}{5} + \frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3,x)
```

```
[Out] x*a^3+a^2*c*x^3+3/5*a*c^2*x^5+1/7*c^3*x^7
```

Maxima [A] time = 1.14035, size = 42, normalized size = 1.2

$$\frac{1}{7}c^3x^7 + \frac{3}{5}ac^2x^5 + a^2cx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x
```

Fricas [A] time = 1.85996, size = 66, normalized size = 1.89

$$\frac{1}{7}x^7c^3 + \frac{3}{5}x^5c^2a + x^3ca^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^3 + 3/5*x^5*c^2*a + x^3*c*a^2 + x*a^3
```

Sympy [A] time = 0.066568, size = 32, normalized size = 0.91

$$a^3x + a^2cx^3 + \frac{3ac^2x^5}{5} + \frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3,x)
```

```
[Out] a**3*x + a**2*c*x**3 + 3*a*c**2*x**5/5 + c**3*x**7/7
```

Giac [A] time = 1.19568, size = 42, normalized size = 1.2

$$\frac{1}{7}c^3x^7 + \frac{3}{5}ac^2x^5 + a^2cx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x
```

3.51 $\int \left(a+cx^2\right)^2 \, dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}acx^3+\frac{c^2x^5}{5}$$

```
[Out] a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5
```

Rubi [A] time = 0.0071073, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2x+\frac{2}{3}acx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)^2 \, dx &= \int \left(a^2+2acx^2+c^2x^4\right) \, dx\\ &= a^2x+\frac{2}{3}acx^3+\frac{c^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.001019, size = 25, normalized size = 1.

$$a^2x+\frac{2}{3}acx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5
```

Maple [A] time = 0.051, size = 22, normalized size = 0.9

$$a^2x+\frac{2\,ax^3c}{3}+\frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2,x)
```

```
[Out] a^2*x+2/3*a*x^3*c+1/5*c^2*x^5
```

Maxima [A] time = 1.23573, size = 28, normalized size = 1.12

$$\frac{1}{5}c^2x^5 + \frac{2}{3}acx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x
```

Fricas [A] time = 1.8333, size = 47, normalized size = 1.88

$$\frac{1}{5}x^5c^2 + \frac{2}{3}x^3ca + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c^2 + 2/3*x^3*c*a + x*a^2
```

Sympy [A] time = 0.083565, size = 22, normalized size = 0.88

$$a^2x + \frac{2acx^3}{3} + \frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*c*x**3/3 + c**2*x**5/5
```

Giac [A] time = 1.31485, size = 28, normalized size = 1.12

$$\frac{1}{5}c^2x^5 + \frac{2}{3}acx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x
```

3.52 $\int \left(a+cx^2\right) dx$

Optimal. Leaf size=12

$$ax+\frac{cx^3}{3}$$

```
[Out] a*x + (c*x^3)/3
```

Rubi [A] time = 0.0019781, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[a + c*x^2,x]
```

```
[Out] a*x + (c*x^3)/3
```

Rubi steps

$$\int \left(a+cx^2\right) dx=ax+\frac{cx^3}{3}$$

Mathematica [A] time = 0.0000331, size = 12, normalized size = 1.

$$ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + c*x^2,x]
```

```
[Out] a*x + (c*x^3)/3
```

Maple [A] time = 0.046, size = 11, normalized size = 0.9

$$ax+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^2+a,x)
```

```
[Out] a*x+1/3*c*x^3
```

Maxima [A] time = 1.15432, size = 14, normalized size = 1.17

$$\frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+a,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + a*x
```

Fricas [A] time = 1.813, size = 23, normalized size = 1.92

$$\frac{1}{3}x^3c + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+a,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*c + x*a
```

Sympy [A] time = 0.070719, size = 8, normalized size = 0.67

$$ax + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**2+a,x)
```

```
[Out] a*x + c*x**3/3
```

Giac [A] time = 1.22671, size = 14, normalized size = 1.17

$$\frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+a,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + a*x
```

3.53 $\int \frac{1}{a+cx^2}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])
```

Rubi [A] time = 0.0050706, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+cx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

Mathematica [A] time = 0.0046259, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])
```

Maple [A] time = 0.047, size = 16, normalized size = 0.7

$$\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a),x)
```

```
[Out] 1/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11155, size = 151, normalized size = 6.29

$$\left[-\frac{\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)}{2\,ac}, \frac{\sqrt{ac}\arctan\left(\frac{\sqrt{ac}x}{a}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a))/(a*c), sqrt(
a*c)*arctan(sqrt(a*c)*x/a)/(a*c)]
```

Sympy [B] time = 0.153677, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ac}}\log\left(-a\sqrt{-\frac{1}{ac}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ac}}\log\left(a\sqrt{-\frac{1}{ac}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*c))*log(-a*sqrt(-1/(a*c)) + x)/2 + sqrt(-1/(a*c))*log(a*sqrt(-1
/(a*c)) + x)/2
```

Giac [A] time = 1.23682, size = 20, normalized size = 0.83

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a),x, algorithm="giac")
```

```
[Out] arctan(c*x/sqrt(a*c))/sqrt(a*c)
```

3.54 $\int \frac{1}{(a+cx^2)^2}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a\left(a+cx^2\right)}$$

```
[Out] x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])
```

Rubi [A] time = 0.009972, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+cx^2\right)^2}\,dx &= \frac{x}{2a\left(a+cx^2\right)}+\frac{\int \frac{1}{a+cx^2}\,dx}{2a}\\ &= \frac{x}{2a\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.026729, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])
```

Maple [A] time = 0.054, size = 36, normalized size = 0.8

$$\frac{x}{2\,a\left(cx^{2}+a\right)}+\frac{1}{2\,a}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^2,x)
```

```
[Out] 1/2*x/a/(c*x^2+a)+1/2/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16386, size = 261, normalized size = 5.8

$$\left[\frac{2\,acx-\left(cx^{2}+a\right)\sqrt{-ac}\log\left(\frac{cx^{2}-2\,\sqrt{-ac}x-a}{cx^{2}+a}\right)}{4\left(a^{2}c^{2}x^{2}+a^{3}c\right)},\frac{acx+\left(cx^{2}+a\right)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}x}{a}\right)}{2\left(a^{2}c^{2}x^{2}+a^{3}c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*c*x - (c*x^2 + a)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*
x^2 + a)))/(a^2*c^2*x^2 + a^3*c), 1/2*(a*c*x + (c*x^2 + a)*sqrt(a*c)*arctan
(sqrt(a*c)*x/a))/(a^2*c^2*x^2 + a^3*c)]
```

Sympy [B] time = 0.392589, size = 78, normalized size = 1.73

$$\frac{x}{2a^{2}+2acx^{2}}-\frac{\sqrt{-\frac{1}{a^{3}c}}\log\left(-a^{2}\sqrt{-\frac{1}{a^{3}c}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^{3}c}}\log\left(a^{2}\sqrt{-\frac{1}{a^{3}c}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**2,x)
```

```
[Out] x/(2*a**2 + 2*a*c*x**2) - sqrt(-1/(a**3*c))*log(-a**2*sqrt(-1/(a**3*c)) + x
)/4 + sqrt(-1/(a**3*c))*log(a**2*sqrt(-1/(a**3*c)) + x)/4
```

Giac [A] time = 1.29076, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}a} + \frac{x}{2\left(cx^2 + a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a) + 1/2*x/((c*x^2 + a)*a)
```

3.55 $\int \frac{1}{\left(a+cx^2\right)^3} \, dx$

Optimal. Leaf size=62

$$\frac{3x}{8a^2\left(a+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}+\frac{x}{4a\left(a+cx^2\right)^2}$$

```
[Out] x/(4*a*(a + c*x^2)^2) + (3*x)/(8*a^2*(a + c*x^2)) + (3*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/(8*a^(5/2)*Sqrt[c])
```

Rubi [A] time = 0.0159544, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 205}

$$\frac{3x}{8a^2\left(a+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}+\frac{x}{4a\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-3),x]
```

```
[Out] x/(4*a*(a + c*x^2)^2) + (3*x)/(8*a^2*(a + c*x^2)) + (3*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/(8*a^(5/2)*Sqrt[c])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+cx^2\right)^3} \, dx &= \frac{x}{4a\left(a+cx^2\right)^2}+\frac{3\int \frac{1}{\left(a+cx^2\right)^2} \, dx}{4a}\\ &= \frac{x}{4a\left(a+cx^2\right)^2}+\frac{3x}{8a^2\left(a+cx^2\right)}+\frac{3\int \frac{1}{a+cx^2} \, dx}{8a^2}\\ &= \frac{x}{4a\left(a+cx^2\right)^2}+\frac{3x}{8a^2\left(a+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0380062, size = 55, normalized size = 0.89

$$\frac{5ax+3cx^3}{8a^2\left(a+cx^2\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-3),x]
```

```
[Out] (5*a*x + 3*c*x^3)/(8*a^2*(a + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(
8*a^(5/2)*Sqrt[c])
```

Maple [A] time = 0.05, size = 51, normalized size = 0.8

$$\frac{x}{4\,a\left(cx^2+a\right)^2}+\frac{3\,x}{8\,a^2\left(cx^2+a\right)}+\frac{3}{8\,a^2}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^3,x)
```

```
[Out] 1/4*x/a/(c*x^2+a)^2+3/8*x/a^2/(c*x^2+a)+3/8/a^2/(a*c)^(1/2)*arctan(x*c/(a*c
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07591, size = 401, normalized size = 6.47

$$\left[\frac{6\,ac^2x^3+10\,a^2cx-3\left(c^2x^4+2\,acx^2+a^2\right)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)}{16\left(a^3c^3x^4+2\,a^4c^2x^2+a^5c\right)},\frac{3\,ac^2x^3+5\,a^2cx+3\left(c^2x^4+2\,acx^2+a^2\right)\sqrt{ac}\arctan}{8\left(a^3c^3x^4+2\,a^4c^2x^2+a^5c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*c^2*x^3 + 10*a^2*c*x - 3*(c^2*x^4 + 2*a*c*x^2 + a^2)*sqrt(-a*c)*
log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a^3*c^3*x^4 + 2*a^4*c^2*x^2
 + a^5*c), 1/8*(3*a*c^2*x^3 + 5*a^2*c*x + 3*(c^2*x^4 + 2*a*c*x^2 + a^2)*sqr
t(a*c)*arctan(sqrt(a*c)*x/a))/(a^3*c^3*x^4 + 2*a^4*c^2*x^2 + a^5*c)]
```

Sympy [A] time = 0.502658, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(-a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16}+\frac{5ax+3cx^3}{8a^4+16a^3cx^2+8a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*c))*log(-a**3*sqrt(-1/(a**5*c)) + x)/16 + 3*sqrt(-1/(a**5*
c))*log(a**3*sqrt(-1/(a**5*c)) + x)/16 + (5*a*x + 3*c*x**3)/(8*a**4 + 16*a*
*3*c*x**2 + 8*a**2*c**2*x**4)
```

Giac [A] time = 1.19606, size = 61, normalized size = 0.98

$$\frac{3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2}+\frac{3\,cx^3+5\,ax}{8\left(cx^2+a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/8*(3*c*x^3 + 5*a*x)/((c*x^2 +
 a)^2*a^2)
```

3.56 $\int \left(a+cx^2\right)^{5/2} \, dx$

Optimal. Leaf size=84

$$\frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}$$

```
[Out] (5*a^2*x*Sqrt[a + c*x^2])/16 + (5*a*x*(a + c*x^2)^(3/2))/24 + (x*(a + c*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*Sqrt[c])
```

Rubi [A] time = 0.0224621, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2),x]
```

```
[Out] (5*a^2*x*Sqrt[a + c*x^2])/16 + (5*a*x*(a + c*x^2)^(3/2))/24 + (x*(a + c*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*Sqrt[c])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+cx^2\right)^{5/2} dx = \frac{1}{6}x\left(a+cx^2\right)^{5/2} + \frac{1}{6}(5a)\int \left(a+cx^2\right)^{3/2} dx$$
$$= \frac{5}{24}ax\left(a+cx^2\right)^{3/2} + \frac{1}{6}x\left(a+cx^2\right)^{5/2} + \frac{1}{8}\left(5a^2\right)\int \sqrt{a+cx^2}\,dx$$
$$= \frac{5}{16}a^2x\sqrt{a+cx^2} + \frac{5}{24}ax\left(a+cx^2\right)^{3/2} + \frac{1}{6}x\left(a+cx^2\right)^{5/2} + \frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{a+cx^2}}dx$$
$$= \frac{5}{16}a^2x\sqrt{a+cx^2} + \frac{5}{24}ax\left(a+cx^2\right)^{3/2} + \frac{1}{6}x\left(a+cx^2\right)^{5/2} + \frac{1}{16}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-cx^2}dx, x, \frac{x}{\sqrt{a+c}}\right.$$
$$= \frac{5}{16}a^2x\sqrt{a+cx^2} + \frac{5}{24}ax\left(a+cx^2\right)^{3/2} + \frac{1}{6}x\left(a+cx^2\right)^{5/2} + \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}$$

Mathematica [A] time = 0.111205, size = 76, normalized size = 0.9

$$\frac{1}{48}\sqrt{a+cx^2}\left(\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{a}+1}} + 33a^2x + 26acx^3 + 8c^2x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(33*a^2*x + 26*a*c*x^3 + 8*c^2*x^5 + (15*a^(5/2)*ArcSinh[(
Sqrt[c]*x)/Sqrt[a]])/(Sqrt[c]*Sqrt[1 + (c*x^2)/a])))/48
```

Maple [A] time = 0.047, size = 66, normalized size = 0.8

$$\frac{x}{6}\left(cx^2+a\right)^{\frac{5}{2}} + \frac{5\,ax}{24}\left(cx^2+a\right)^{\frac{3}{2}} + \frac{5\,a^2x}{16}\sqrt{cx^2+a} + \frac{5\,a^3}{16}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2),x)
```

```
[Out] 1/6*x*(c*x^2+a)^(5/2)+5/24*a*x*(c*x^2+a)^(3/2)+5/16*a^2*x*(c*x^2+a)^(1/2)+5
/16*a^3/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.33734, size = 347, normalized size = 4.13

$$\left[\frac{15\,a^3\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(8\,c^3x^5+26\,ac^2x^3+33\,a^2cx\right)\sqrt{cx^2+a}}{96\,c},-\frac{15\,a^3\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-\left(8\,c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(
8*c^3*x^5 + 26*a*c^2*x^3 + 33*a^2*c*x)*sqrt(c*x^2 + a))/c, -1/48*(15*a^3*sq
rt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (8*c^3*x^5 + 26*a*c^2*x^3 + 33*
a^2*c*x)*sqrt(c*x^2 + a))/c]
```

Sympy [A] time = 5.60374, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{cx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}cx^3\sqrt{1+\frac{cx^2}{a}}}{24}+\frac{\sqrt{a}c^2x^5\sqrt{1+\frac{cx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2),x)
```

```
[Out] 11*a**(5/2)*x*sqrt(1 + c*x**2/a)/16 + 13*a**(3/2)*c*x**3*sqrt(1 + c*x**2/a)
/24 + sqrt(a)*c**2*x**5*sqrt(1 + c*x**2/a)/6 + 5*a**3*asinh(sqrt(c)*x/sqrt(
a))/(16*sqrt(c))
```

Giac [A] time = 1.21088, size = 85, normalized size = 1.01

$$-\frac{5\,a^3\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{16\,\sqrt{c}}+\frac{1}{48}\left(2\left(4\,c^2x^2+13\,ac\right)x^2+33\,a^2\right)\sqrt{cx^2+a}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + 1/48*(2*(4*c^2*x
^2 + 13*a*c)*x^2 + 33*a^2)*sqrt(c*x^2 + a)*x
```

3.57 $\int \left(a+cx^2\right)^{3/2} dx$

Optimal. Leaf size=65

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}+\frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}$$

```
[Out] (3*a*x*Sqrt[a + c*x^2])/8 + (x*(a + c*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[
c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c])
```

Rubi [A] time = 0.0143466, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}+\frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2),x]
```


```
[Out] (3*a*x*Sqrt[a + c*x^2])/8 + (x*(a + c*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[
c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c])
```


Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)^{3/2} dx &= \frac{1}{4}x\left(a+cx^2\right)^{3/2}+\frac{1}{4}(3a)\int \sqrt{a+cx^2}\,dx\\ &= \frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{a+cx^2}}dx\\ &= \frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-cx^2}dx,x,\frac{x}{\sqrt{a+cx^2}}\right)\\ &= \frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0888968, size = 65, normalized size = 1.

$$\frac{1}{8}\sqrt{a+cx^2}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{a}+1}}+5ax+2cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(5*a*x + 2*c*x^3 + (3*a^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]
)/(Sqrt[c]*Sqrt[1 + (c*x^2)/a])))/8
```

Maple [A] time = 0.051, size = 51, normalized size = 0.8

$$\frac{x}{4}\left(cx^2+a\right)^{\frac{3}{2}}+\frac{3\,ax}{8}\sqrt{cx^2+a}+\frac{3\,a^2}{8}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2),x)
```

```
[Out] 1/4*x*(c*x^2+a)^(3/2)+3/8*a*x*(c*x^2+a)^(1/2)+3/8*a^2/c^(1/2)*ln(x*c^(1/2)+
(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.36263, size = 294, normalized size = 4.52

$$\left[\frac{3\,a^2\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(2\,c^2x^3+5\,acx\right)\sqrt{cx^2+a}}{16\,c}, -\frac{3\,a^2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-\left(2\,c^2x^3+5\,acx\right)\sqrt{c}}{8\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(2
*c^2*x^3 + 5*a*c*x)*sqrt(c*x^2 + a))/c, -1/8*(3*a^2*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) - (2*c^2*x^3 + 5*a*c*x)*sqrt(c*x^2 + a))/c]
```

Sympy [A] time = 3.8631, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{cx^2}{a}}}{8}+\frac{\sqrt{a}cx^3\sqrt{1+\frac{cx^2}{a}}}{4}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x*sqrt(1 + c*x**2/a)/8 + sqrt(a)*c*x**3*sqrt(1 + c*x**2/a)/4 + 3
*a**2*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c))
```

Giac [A] time = 1.22841, size = 66, normalized size = 1.02

$$\frac{1}{8}\left(2\,cx^2+5\,a\right)\sqrt{cx^2+a}x-\frac{3\,a^2\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*c*x^2 + 5*a)*sqrt(c*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(c)*x + sqrt(c
*x^2 + a)))/sqrt(c)
```

3.58 $\int \sqrt{a+cx^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}x\sqrt{a+cx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

```
[Out] (x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]
)
```

Rubi [A] time = 0.0089727, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{1}{2}x\sqrt{a+cx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2],x]
```

```
[Out] (x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+cx^2}\,dx &= \frac{1}{2}x\sqrt{a+cx^2}+\frac{1}{2}a\int \frac{1}{\sqrt{a+cx^2}}\,dx\\ &= \frac{1}{2}x\sqrt{a+cx^2}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)\\ &= \frac{1}{2}x\sqrt{a+cx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0205114, size = 49, normalized size = 1.07

$$\frac{1}{2}x\sqrt{a+cx^2}+\frac{a\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2],x]
```

```
[Out] (x*Sqrt[a + c*x^2])/2 + (a*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(2*Sqrt[c])
```

Maple [A] time = 0.047, size = 36, normalized size = 0.8

$$\frac{x}{2}\sqrt{cx^2+a}+\frac{a}{2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.38939, size = 232, normalized size = 5.04

$$\left[\frac{2\sqrt{cx^2+a}cx+a\sqrt{c}\log\left(-2cx^2-2\sqrt{cx^2+a}\sqrt{c}x-a\right)}{4c}, \frac{\sqrt{cx^2+a}cx-a\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)}{2c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*x^2 + a)*c*x + a*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a))/c, 1/2*(sqrt(c*x^2 + a)*c*x - a*sqrt(-c)*arctan(sqrt(-c)*x/sq
rt(c*x^2 + a)))/c]
```

Sympy [A] time = 2.71647, size = 41, normalized size = 0.89

$$\frac{\sqrt{a}x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + c*x**2/a)/2 + a*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c))
```

Giac [A] time = 1.24465, size = 50, normalized size = 1.09

$$\frac{1}{2}\sqrt{cx^2+a}x-\frac{a\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*x - 1/2*a*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c
)
```

3.59 $\int \frac{1}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]
```

Rubi [A] time = 0.0052679, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + c*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+cx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0060221, size = 25, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + c*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]
```

Maple [A] time = 0.046, size = 21, normalized size = 0.8

$$\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(1/2),x)
```

```
[Out] ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13793, size = 153, normalized size = 6.12

$$\left[\frac{\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)}{2\,\sqrt{c}},-\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a)/sqrt(c), -sqrt(-c)*arc
tan(sqrt(-c)*x/sqrt(c*x^2 + a))/c]
```

Sympy [A] time = 1.59242, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(1/2),x)
```

```
[Out] asinh(sqrt(c)*x/sqrt(a))/sqrt(c)
```

Giac [A] time = 1.3434, size = 31, normalized size = 1.24

$$-\frac{\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c)
```

3.60 $\int \frac{1}{(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt{a+cx^2}}$$

```
[Out] x/(a*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0019738, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {191}

$$\frac{x}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{x}{a\sqrt{a+cx^2}}$$

Mathematica [A] time = 0.0041616, size = 16, normalized size = 1.

$$\frac{x}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + c*x^2])
```

Maple [A] time = 0.047, size = 15, normalized size = 0.9

$$\frac{x}{a}\frac{1}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(3/2),x)
```

```
[Out] x/a/(c*x^2+a)^(1/2)
```

Maxima [A] time = 1.05934, size = 19, normalized size = 1.19

$$\frac{x}{\sqrt{cx^2 + a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(c*x^2 + a)*a)
```

Fricas [A] time = 2.13287, size = 47, normalized size = 2.94

$$\frac{\sqrt{cx^2 + a}x}{acx^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2 + a)*x/(a*c*x^2 + a^2)
```

Sympy [A] time = 0.848895, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1 + \frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(3/2),x)
```

```
[Out] x/(a**(3/2)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.23389, size = 19, normalized size = 1.19

$$\frac{x}{\sqrt{cx^2 + a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(c*x^2 + a)*a)
```

3.61 $\int \frac{1}{\left(a+cx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=39

$$\frac{2x}{3a^2\sqrt{a+cx^2}}+\frac{x}{3a\left(a+cx^2\right)^{3/2}}$$

```
[Out] x/(3*a*(a + c*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0056857, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{2x}{3a^2\sqrt{a+cx^2}}+\frac{x}{3a\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-5/2),x]
```

```
[Out] x/(3*a*(a + c*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + c*x^2])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+cx^2\right)^{5/2}} \, dx &= \frac{x}{3a\left(a+cx^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(a+cx^2\right)^{3/2}} \, dx}{3a}\\ &= \frac{x}{3a\left(a+cx^2\right)^{3/2}}+\frac{2x}{3a^2\sqrt{a+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0085884, size = 29, normalized size = 0.74

$$\frac{x\left(3a+2cx^2\right)}{3a^2\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-5/2),x]
```

```
[Out] (x*(3*a + 2*c*x^2))/(3*a^2*(a + c*x^2)^(3/2))
```

Maple [A] time = 0.049, size = 26, normalized size = 0.7

$$\frac{x\left(2\,c{x}^{2}+3\,a\right)}{3\,{a}^{2}}\left(c{x}^{2}+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x*(2*c*x^2+3*a)/(c*x^2+a)^(3/2)/a^2
```

Maxima [A] time = 1.13467, size = 42, normalized size = 1.08

$$\frac{2\,x}{3\,\sqrt{cx^{2}+a}a^{2}}+\frac{x}{3\,\left(cx^{2}+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(c*x^2 + a)*a^2) + 1/3*x/((c*x^2 + a)^(3/2)*a)
```

Fricas [A] time = 2.26115, size = 99, normalized size = 2.54

$$\frac{\left(2\,cx^{3}+3\,ax\right)\sqrt{cx^{2}+a}}{3\,\left(a^{2}c^{2}x^{4}+2\,a^{3}cx^{2}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*c*x^3 + 3*a*x)*sqrt(c*x^2 + a)/(a^2*c^2*x^4 + 2*a^3*c*x^2 + a^4)
```

Sympy [B] time = 1.18182, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^{2}}{a}}+3a^{\frac{5}{2}}cx^{2}\sqrt{1+\frac{cx^{2}}{a}}}+\frac{2cx^{3}}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^{2}}{a}}+3a^{\frac{5}{2}}cx^{2}\sqrt{1+\frac{cx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(5/2),x)
```

```
[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*x**2/a)
) + 2*c*x**3/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*
x**2/a))
```

Giac [A] time = 1.23899, size = 36, normalized size = 0.92

$$\frac{x\left(\frac{2\,cx^2}{a^2}+\frac{3}{a}\right)}{3\left(cx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*c*x^2/a^2 + 3/a)/(c*x^2 + a)^(3/2)
```

3.62 $\int \frac{1}{\left(a+cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=58

$$\frac{8x}{15a^3\sqrt{a+cx^2}}+\frac{4x}{15a^2\left(a+cx^2\right)^{3/2}}+\frac{x}{5a\left(a+cx^2\right)^{5/2}}$$

```
[Out] x/(5*a*(a + c*x^2)^(5/2)) + (4*x)/(15*a^2*(a + c*x^2)^(3/2)) + (8*x)/(15*a^
3*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0104298, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{8x}{15a^3\sqrt{a+cx^2}}+\frac{4x}{15a^2\left(a+cx^2\right)^{3/2}}+\frac{x}{5a\left(a+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-7/2),x]
```

```
[Out] x/(5*a*(a + c*x^2)^(5/2)) + (4*x)/(15*a^2*(a + c*x^2)^(3/2)) + (8*x)/(15*a^
3*Sqrt[a + c*x^2])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+cx^2\right)^{7/2}}\,dx &= \frac{x}{5a\left(a+cx^2\right)^{5/2}}+\frac{4\int \frac{1}{\left(a+cx^2\right)^{5/2}}\,dx}{5a}\\
&= \frac{x}{5a\left(a+cx^2\right)^{5/2}}+\frac{4x}{15a^2\left(a+cx^2\right)^{3/2}}+\frac{8\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{15a^2}\\
&= \frac{x}{5a\left(a+cx^2\right)^{5/2}}+\frac{4x}{15a^2\left(a+cx^2\right)^{3/2}}+\frac{8x}{15a^3\sqrt{a+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0109272, size = 40, normalized size = 0.69

$$\frac{x\left(15a^2+20acx^2+8c^2x^4\right)}{15a^3\left(a+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-7/2),x]
```

```
[Out] (x*(15*a^2 + 20*a*c*x^2 + 8*c^2*x^4))/(15*a^3*(a + c*x^2)^(5/2))
```

Maple [A] time = 0.048, size = 37, normalized size = 0.6

$$\frac{x\left(8\,c^2x^4+20\,ax^2c+15\,a^2\right)}{15\,a^3}\left(cx^2+a\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(7/2),x)
```

```
[Out] 1/15*x*(8*c^2*x^4+20*a*c*x^2+15*a^2)/(c*x^2+a)^(5/2)/a^3
```

Maxima [A] time = 1.0683, size = 62, normalized size = 1.07

$$\frac{8\,x}{15\,\sqrt{cx^2+a}a^3}+\frac{4\,x}{15\left(cx^2+a\right)^{\frac{3}{2}}a^2}+\frac{x}{5\left(cx^2+a\right)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*x/(sqrt(c*x^2 + a)*a^3) + 4/15*x/((c*x^2 + a)^(3/2)*a^2) + 1/5*x/((c*x
^2 + a)^(5/2)*a)
```

Fricas [A] time = 2.25931, size = 146, normalized size = 2.52

$$\frac{\left(8\,c^2x^5+20\,acx^3+15\,a^2x\right)\sqrt{cx^2+a}}{15\left(a^3c^3x^6+3\,a^4c^2x^4+3\,a^5cx^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*c^2*x^5 + 20*a*c*x^3 + 15*a^2*x)*sqrt(c*x^2 + a)/(a^3*c^3*x^6 + 3*a
^4*c^2*x^4 + 3*a^5*c*x^2 + a^6)
```

Sympy [B] time = 2.28901, size = 413, normalized size = 7.12

$$\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}+\frac{35}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(7/2),x)
```

```
[Out] 15*a**5*x/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c
*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x*
*6*sqrt(1 + c*x**2/a)) + 35*a**4*c*x**3/(15*a**(17/2)*sqrt(1 + c*x**2/a) +
45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*
x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a)) + 28*a**3*c**2*x**5/(1
5*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 4
5*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 +
c*x**2/a)) + 8*a**2*c**3*x**7/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/
2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) +
15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.24094, size = 55, normalized size = 0.95

$$\frac{\left(4\,x^2\left(\frac{2\,c^2x^2}{a^3}+\frac{5\,c}{a^2}\right)+\frac{15}{a}\right)x}{15\left(cx^2+a\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(4*x^2*(2*c^2*x^2/a^3 + 5*c/a^2) + 15/a)*x/(c*x^2 + a)^(5/2)
```

3.63 $\int \frac{1}{\left(a+cx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=77

$$\frac{16x}{35a^4\sqrt{a+cx^2}}+\frac{8x}{35a^3\left(a+cx^2\right)^{3/2}}+\frac{6x}{35a^2\left(a+cx^2\right)^{5/2}}+\frac{x}{7a\left(a+cx^2\right)^{7/2}}$$

```
[Out] x/(7*a*(a + c*x^2)^(7/2)) + (6*x)/(35*a^2*(a + c*x^2)^(5/2)) + (8*x)/(35*a^
3*(a + c*x^2)^(3/2)) + (16*x)/(35*a^4*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0159789, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{16x}{35a^4\sqrt{a+cx^2}}+\frac{8x}{35a^3\left(a+cx^2\right)^{3/2}}+\frac{6x}{35a^2\left(a+cx^2\right)^{5/2}}+\frac{x}{7a\left(a+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-9/2),x]
```

```
[Out] x/(7*a*(a + c*x^2)^(7/2)) + (6*x)/(35*a^2*(a + c*x^2)^(5/2)) + (8*x)/(35*a^
3*(a + c*x^2)^(3/2)) + (16*x)/(35*a^4*Sqrt[a + c*x^2])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+cx^2\right)^{9/2}}\,dx &= \frac{x}{7a\left(a+cx^2\right)^{7/2}}+\frac{6\int \frac{1}{\left(a+cx^2\right)^{7/2}}\,dx}{7a}\\
&= \frac{x}{7a\left(a+cx^2\right)^{7/2}}+\frac{6x}{35a^2\left(a+cx^2\right)^{5/2}}+\frac{24\int \frac{1}{\left(a+cx^2\right)^{5/2}}\,dx}{35a^2}\\
&= \frac{x}{7a\left(a+cx^2\right)^{7/2}}+\frac{6x}{35a^2\left(a+cx^2\right)^{5/2}}+\frac{8x}{35a^3\left(a+cx^2\right)^{3/2}}+\frac{16\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{35a^3}\\
&= \frac{x}{7a\left(a+cx^2\right)^{7/2}}+\frac{6x}{35a^2\left(a+cx^2\right)^{5/2}}+\frac{8x}{35a^3\left(a+cx^2\right)^{3/2}}+\frac{16x}{35a^4\sqrt{a+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0142146, size = 51, normalized size = 0.66

$$\frac{x\left(70a^2cx^2+35a^3+56ac^2x^4+16c^3x^6\right)}{35a^4\left(a+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-9/2),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*c*x^2 + 56*a*c^2*x^4 + 16*c^3*x^6))/(35*a^4*(a + c*x^2)
^(7/2))
```

Maple [A] time = 0.05, size = 48, normalized size = 0.6

$$\frac{x\left(16\,c^3x^6+56\,ac^2x^4+70\,a^2cx^2+35\,a^3\right)}{35\,a^4}\left(cx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(9/2),x)
```

```
[Out] 1/35*x*(16*c^3*x^6+56*a*c^2*x^4+70*a^2*c*x^2+35*a^3)/(c*x^2+a)^(7/2)/a^4
```

Maxima [A] time = 1.13711, size = 82, normalized size = 1.06

$$\frac{16\,x}{35\,\sqrt{cx^2+a}a^4}+\frac{8\,x}{35\left(cx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,x}{35\left(cx^2+a\right)^{\frac{5}{2}}a^2}+\frac{x}{7\left(cx^2+a\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*x/(sqrt(c*x^2 + a)*a^4) + 8/35*x/((c*x^2 + a)^(3/2)*a^3) + 6/35*x/((c
*x^2 + a)^(5/2)*a^2) + 1/7*x/((c*x^2 + a)^(7/2)*a)
```

Fricas [A] time = 2.30588, size = 192, normalized size = 2.49

$$\frac{\left(16\,c^3x^7+56\,ac^2x^5+70\,a^2cx^3+35\,a^3x\right)\sqrt{cx^2+a}}{35\left(a^4c^4x^8+4\,a^5c^3x^6+6\,a^6c^2x^4+4\,a^7cx^2+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*c^3*x^7 + 56*a*c^2*x^5 + 70*a^2*c*x^3 + 35*a^3*x)*sqrt(c*x^2 + a)/
(a^4*c^4*x^8 + 4*a^5*c^3*x^6 + 6*a^6*c^2*x^4 + 4*a^7*c*x^2 + a^8)
```

Sympy [B] time = 4.73207, size = 1265, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(9/2),x)
```

```
[Out] 35*a**14*x/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 +
 c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**
3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 21
0*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1
+ c*x**2/a)) + 175*a**13*c*x**3/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(
35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a
) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sq
rt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/
2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 371*a**12*c**2*x**5/(35*a**(37/2)*sqrt(
1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**
2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 52
5*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1
+ c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 429*a**11*c**3*
x**7/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**
2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6
*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(
27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x*
*2/a)) + 286*a**10*c**4*x**9/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/
2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) +
 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(
1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*
c**6*x**12*sqrt(1 + c*x**2/a)) + 104*a**9*c**5*x**11/(35*a**(37/2)*sqrt(1 +
 c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x
**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a
**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c
*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 16*a**8*c**6*x**13
/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a)
 + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqr
t(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2
)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a
))
```

Giac [A] time = 1.19661, size = 74, normalized size = 0.96

$$\frac{\left(2\left(4x^2\left(\frac{2c^3x^2}{a^4}+\frac{7c^2}{a^3}\right)+\frac{35c}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\left(cx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(2*(4*x^2*(2*c^3*x^2/a^4 + 7*c^2/a^3) + 35*c/a^2)*x^2 + 35/a)*x/(c*x^2
 + a)^(7/2)
```

3.64 $\int \left(4+12x+9x^2\right)^{3/2} dx$

Optimal. Leaf size=23

$$\frac{1}{12}(3x+2)\left(9x^2+12x+4\right)^{3/2}$$

```
[Out] ((2 + 3*x)*(4 + 12*x + 9*x^2)^(3/2))/12
```

Rubi [A] time = 0.0028427, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {609}

$$\frac{1}{12}(3x+2)\left(9x^2+12x+4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 12*x + 9*x^2)^(3/2),x]
```

```
[Out] ((2 + 3*x)*(4 + 12*x + 9*x^2)^(3/2))/12
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \left(4+12x+9x^2\right)^{3/2} dx = \frac{1}{12}(2+3x)\left(4+12x+9x^2\right)^{3/2}$$

Mathematica [A] time = 0.0087792, size = 20, normalized size = 0.87

$$\frac{1}{12}(3x+2)\left((3x+2)^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 12*x + 9*x^2)^(3/2),x]
```

```
[Out] ((2 + 3*x)*((2 + 3*x)^2)^(3/2))/12
```

Maple [A] time = 0.071, size = 35, normalized size = 1.5

$$\frac{x\left(27x^3+72x^2+72x+32\right)}{4\,(2+3x)^3}\left((2+3x)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2+12*x+4)^(3/2),x)
```

```
[Out] 1/4*x*(27*x^3+72*x^2+72*x+32)*((2+3*x)^2)^(3/2)/(2+3*x)^3
```

Maxima [A] time = 1.70056, size = 41, normalized size = 1.78

$$\frac{1}{4}\left(9x^2+12x+4\right)^{\frac{3}{2}}x+\frac{1}{6}\left(9x^2+12x+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 12*x + 4)^(3/2)*x + 1/6*(9*x^2 + 12*x + 4)^(3/2)
```

Fricas [A] time = 2.08727, size = 46, normalized size = 2.

$$\frac{27}{4}x^4+18x^3+18x^2+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(3/2),x, algorithm="fricas")
```

```
[Out] 27/4*x^4 + 18*x^3 + 18*x^2 + 8*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(9x^2+12x+4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2+12*x+4)**(3/2),x)
```

```
[Out] Integral((9*x**2 + 12*x + 4)**(3/2), x)
```

Giac [B] time = 1.20392, size = 69, normalized size = 3.

$$\frac{27}{4}x^4\mathrm{sgn}\left(3x+2\right)+18x^3\mathrm{sgn}\left(3x+2\right)+18x^2\mathrm{sgn}\left(3x+2\right)+8x\mathrm{sgn}\left(3x+2\right)+\frac{4}{3}\mathrm{sgn}\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(3/2),x, algorithm="giac")
```

```
[Out] 27/4*x^4*sgn(3*x + 2) + 18*x^3*sgn(3*x + 2) + 18*x^2*sgn(3*x + 2) + 8*x*sgn
(3*x + 2) + 4/3*sgn(3*x + 2)
```

3.65 $\int \sqrt{4+12x+9x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{6}(3x+2)\sqrt{9x^2+12x+4}$$

```
[Out] ((2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])/6
```

Rubi [A] time = 0.0031282, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {609}

$$\frac{1}{6}(3x+2)\sqrt{9x^2+12x+4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])/6
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{4+12x+9x^2}\,dx = \frac{1}{6}(2+3x)\sqrt{4+12x+9x^2}$$

Mathematica [A] time = 0.0054391, size = 25, normalized size = 1.09

$$\frac{x\sqrt{(3x+2)^2}(3x+4)}{6x+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] (x*Sqrt[(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)
```

Maple [A] time = 0.069, size = 25, normalized size = 1.1

$$\frac{x(3x+4)}{4+6x}\sqrt{(2+3x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2+12*x+4)^(1/2),x)
```

```
[Out] 1/2*x*(3*x+4)*((2+3*x)^2)^(1/2)/(2+3*x)
```

Maxima [A] time = 1.76622, size = 41, normalized size = 1.78

$$\frac{1}{2}\sqrt{9x^2+12x+4}x+\frac{1}{3}\sqrt{9x^2+12x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 + 12*x + 4)*x + 1/3*sqrt(9*x^2 + 12*x + 4)
```

Fricas [A] time = 2.13546, size = 20, normalized size = 0.87

$$\frac{3}{2}x^2+2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*x^2 + 2*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{9x^2+12x+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2+12*x+4)**(1/2),x)
```

```
[Out] Integral(sqrt(9*x**2 + 12*x + 4), x)
```

Giac [A] time = 1.24368, size = 35, normalized size = 1.52

$$\frac{1}{2}\left(3x^2+4x\right)\operatorname{sgn}(3x+2)+\frac{2}{3}\operatorname{sgn}(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(3*x^2 + 4*x)*sgn(3*x + 2) + 2/3*sgn(3*x + 2)
```

3.66 $\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx$

Optimal. Leaf size=29

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{9x^2+12x+4}}$$

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[4 + 12*x + 9*x^2])
```

Rubi [A] time = 0.0048338, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {608, 31}

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{9x^2+12x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[4 + 12*x + 9*x^2])
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx &= \frac{(6+9x)\int \frac{1}{6+9x}\,dx}{\sqrt{4+12x+9x^2}}\\ &= \frac{(2+3x)\log(2+3x)}{3\sqrt{4+12x+9x^2}}\end{aligned}$$

Mathematica [A] time = 0.0063044, size = 26, normalized size = 0.9

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[(2 + 3*x)^2])
```

Maple [A] time = 0.126, size = 23, normalized size = 0.8

$$\frac{(2+3x)\ln(2+3x)}{3}\frac{1}{\sqrt{(2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2+12*x+4)^(1/2),x)
```

```
[Out] 1/3*(2+3*x)*ln(2+3*x)/((2+3*x)^2)^(1/2)
```

Maxima [A] time = 1.66952, size = 8, normalized size = 0.28

$$\frac{1}{3}\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log(x + 2/3)
```

Fricas [A] time = 2.18526, size = 24, normalized size = 0.83

$$\frac{1}{3}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{9x^2+12x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2+12*x+4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 + 12*x + 4), x)
```

Giac [A] time = 1.22704, size = 34, normalized size = 1.17

$$\frac{\log(|3x+2||\mathrm{sgn}(3x+2)|)}{3\,\mathrm{sgn}(3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(3*x + 2)*abs(sgn(3*x + 2)))/sgn(3*x + 2)
```

3.67 $\int \frac{1}{\left(4+12x+9x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{6(3x+2)\sqrt{9x^2+12x+4}}$$

```
[Out] -1/(6*(2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])
```

Rubi [A] time = 0.0026011, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {607}

$$-\frac{1}{6(3x+2)\sqrt{9x^2+12x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 12*x + 9*x^2)^(-3/2),x]

[Out] -1/(6*(2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(4+12x+9x^2\right)^{3/2}}\,dx=-\frac{1}{6(2+3x)\sqrt{4+12x+9x^2}}$$

Mathematica [A] time = 0.0056801, size = 20, normalized size = 0.8

$$-\frac{3x+2}{6\left((3x+2)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 12*x + 9*x^2)^(-3/2),x]

[Out] -(2 + 3*x)/(6*((2 + 3*x)^2)^(3/2))
```

Maple [A] time = 0.076, size = 17, normalized size = 0.7

$$-\frac{2+3\,x}{6}\left((2+3\,x)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2+12*x+4)^(3/2),x)
```

```
[Out] -1/6*(2+3*x)/((2+3*x)^2)^(3/2)
```

Maxima [A] time = 1.6986, size = 12, normalized size = 0.48

$$-\frac{1}{6(3x+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6/(3*x + 2)^2
```

Fricas [A] time = 2.05615, size = 34, normalized size = 1.36

$$-\frac{1}{6\left(9x^2+12x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6/(9*x^2 + 12*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(9x^2+12x+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2+12*x+4)**(3/2),x)
```

```
[Out] Integral((9*x**2 + 12*x + 4)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.68 $\int \sqrt{4-12x+9x^2}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{6}(2-3x)\sqrt{9x^2-12x+4}$$

```
[Out] -((2 - 3*x)*Sqrt[4 - 12*x + 9*x^2])/6
```

Rubi [A] time = 0.0029693, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {609}

$$-\frac{1}{6}(2-3x)\sqrt{9x^2-12x+4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 - 12*x + 9*x^2],x]

[Out] -((2 - 3*x)*Sqrt[4 - 12*x + 9*x^2])/6
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{4-12x+9x^2}\,dx = -\frac{1}{6}(2-3x)\sqrt{4-12x+9x^2}$$

Mathematica [A] time = 0.0112019, size = 25, normalized size = 1.09

$$\frac{\sqrt{(2-3x)^2}x(3x-4)}{6x-4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 - 12*x + 9*x^2],x]

[Out] (Sqrt[(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)
```

Maple [A] time = 0.075, size = 25, normalized size = 1.1

$$\frac{x(3x-4)}{-4+6x}\sqrt{(-2+3x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-2+3*x)^2)^(1/2),x)
```

```
[Out] 1/2*x*(3*x-4)*((-2+3*x)^2)^(1/2)/(-2+3*x)
```

Maxima [A] time = 1.69923, size = 41, normalized size = 1.78

$$\frac{1}{2}\sqrt{9x^2-12x+4}x-\frac{1}{3}\sqrt{9x^2-12x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 - 12*x + 4)*x - 1/3*sqrt(9*x^2 - 12*x + 4)
```

Fricas [A] time = 2.03042, size = 20, normalized size = 0.87

$$\frac{3}{2}x^2-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*x^2 - 2*x
```

Sympy [A] time = 0.090402, size = 8, normalized size = 0.35

$$\frac{3x^2}{2}-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2+3*x)**2)**(1/2),x)
```

```
[Out] 3*x**2/2 - 2*x
```

Giac [A] time = 1.25786, size = 35, normalized size = 1.52

$$\frac{1}{2}\left(3x^2-4x\right)\operatorname{sgn}(3x-2)+\frac{2}{3}\operatorname{sgn}(3x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(3*x^2 - 4*x)*sgn(3*x - 2) + 2/3*sgn(3*x - 2)
```

3.69 $\int \frac{1}{\sqrt{4-12x+9x^2}} \, dx$

Optimal. Leaf size=29

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{9x^2-12x+4}}$$

```
[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[4 - 12*x + 9*x^2])
```

Rubi [A] time = 0.004519, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {608, 31}

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{9x^2-12x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 - 12*x + 9*x^2],x]
```

```
[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[4 - 12*x + 9*x^2])
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{4-12x+9x^2}} \, dx &= \frac{(-6+9x)\int \frac{1}{-6+9x} \, dx}{\sqrt{4-12x+9x^2}} \\ &= -\frac{(2-3x)\log(2-3x)}{3\sqrt{4-12x+9x^2}} \end{aligned}$$

Mathematica [A] time = 0.0107827, size = 26, normalized size = 0.9

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{(2-3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 - 12*x + 9*x^2],x]
```

```
[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[(2 - 3*x)^2])
```

Maple [A] time = 0.125, size = 23, normalized size = 0.8

$$\frac{(-2+3x)\ln(-2+3x)}{3}\frac{1}{\sqrt{(-2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-2+3*x)^2)^(1/2),x)
```

```
[Out] 1/3/((-2+3*x)^2)^(1/2)*(-2+3*x)*ln(-2+3*x)
```

Maxima [A] time = 1.67276, size = 8, normalized size = 0.28

$$\frac{1}{3}\log\left(x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log(x - 2/3)
```

Fricas [A] time = 2.06614, size = 24, normalized size = 0.83

$$\frac{1}{3}\log(3x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x - 2)
```

Sympy [A] time = 0.09535, size = 7, normalized size = 0.24

$$\frac{\log(3x-2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-2+3*x)**2)**(1/2),x)
```

```
[Out] log(3*x - 2)/3
```

Giac [A] time = 1.62468, size = 20, normalized size = 0.69

$$\frac{1}{3}\log(|3x-2|)\operatorname{sgn}(3x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(3*x - 2))*sgn(3*x - 2)
```

3.70 $\int \sqrt{-4+12x-9x^2}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{6}(2-3x)\sqrt{-9x^2+12x-4}$$

```
[Out] -((2 - 3*x)*Sqrt[-4 + 12*x - 9*x^2])/6
```

Rubi [A] time = 0.0026217, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {609}

$$-\frac{1}{6}(2-3x)\sqrt{-9x^2+12x-4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-4 + 12*x - 9*x^2],x]

[Out] -((2 - 3*x)*Sqrt[-4 + 12*x - 9*x^2])/6
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{-4+12x-9x^2}\,dx = -\frac{1}{6}(2-3x)\sqrt{-4+12x-9x^2}$$

Mathematica [A] time = 0.0076881, size = 27, normalized size = 1.17

$$\frac{\sqrt{-(2-3x)^2}x(3x-4)}{6x-4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-4 + 12*x - 9*x^2],x]

[Out] (Sqrt[-(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)
```

Maple [A] time = 0.066, size = 27, normalized size = 1.2

$$\frac{x\,(3\,x-4)}{-4+6\,x}\sqrt{-\left(-2+3\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-2+3*x)^2)^(1/2),x)
```

```
[Out] 1/2*x*(3*x-4)*(-(-2+3*x)^2)^(1/2)/(-2+3*x)
```

Maxima [A] time = 1.69619, size = 41, normalized size = 1.78

$$\frac{1}{2}\sqrt{-9x^2+12x-4}x-\frac{1}{3}\sqrt{-9x^2+12x-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-9*x^2 + 12*x - 4)*x - 1/3*sqrt(-9*x^2 + 12*x - 4)
```

Fricas [C] time = 2.04588, size = 26, normalized size = 1.13

$$\frac{3}{2}ix^2-2ix$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*I*x^2 - 2*I*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-(3x-2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(3*x - 2)**2), x)
```

Giac [C] time = 1.33137, size = 35, normalized size = 1.52

$$-\frac{1}{2}i\left(3x^2-4x\right)\operatorname{sgn}(-3x+2)-\frac{2}{3}i\operatorname{sgn}(-3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*I*(3*x^2 - 4*x)*sgn(-3*x + 2) - 2/3*I*sgn(-3*x + 2)
```

3.71 $\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-9x^2+12x-4}}$$

```
[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[-4 + 12*x - 9*x^2])
```

Rubi [A] time = 0.0047919, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {608, 31}

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-9x^2+12x-4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[-4 + 12*x - 9*x^2])
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx &= \frac{(6-9x)\int \frac{1}{6-9x}\,dx}{\sqrt{-4+12x-9x^2}}\\ &= -\frac{(2-3x)\log(2-3x)}{3\sqrt{-4+12x-9x^2}}\end{aligned}$$

Mathematica [A] time = 0.0069721, size = 28, normalized size = 0.97

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-(2-3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[-(2 - 3*x)^2])
```

Maple [A] time = 0.11, size = 25, normalized size = 0.9

$$\frac{(-2+3x)\ln(-2+3x)}{3}\frac{1}{\sqrt{-(-2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(-2+3*x)^2)^(1/2),x)
```

```
[Out] 1/3/(-(-2+3*x)^2)^(1/2)*(-2+3*x)*ln(-2+3*x)
```

Maxima [C] time = 1.63142, size = 8, normalized size = 0.28

$$\frac{1}{3}i\log\left(x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*I*log(x - 2/3)
```

Fricas [C] time = 2.10309, size = 28, normalized size = 0.97

$$-\frac{1}{3}i\log\left(x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*I*log(x - 2/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-(3x-2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-(3*x - 2)**2), x)
```

Giac [C] time = 1.20126, size = 31, normalized size = 1.07

$$\frac{i\log\left((-3ix+2i)\operatorname{sgn}(-3x+2)\right)}{3\operatorname{sgn}(-3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*I*log((-3*I*x + 2*I)*sgn(-3*x + 2))/sgn(-3*x + 2)
```

3.72 $\int \sqrt{-4-12x-9x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{6}(3x+2)\sqrt{-9x^2-12x-4}$$

```
[Out] ((2 + 3*x)*Sqrt[-4 - 12*x - 9*x^2])/6
```

Rubi [A] time = 0.0025759, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {609}

$$\frac{1}{6}(3x+2)\sqrt{-9x^2-12x-4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[-4 - 12*x - 9*x^2])/6
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{-4-12x-9x^2}\,dx = \frac{1}{6}(2+3x)\sqrt{-4-12x-9x^2}$$

Mathematica [A] time = 0.0057645, size = 27, normalized size = 1.17

$$\frac{x\sqrt{-(3x+2)^2}(3x+4)}{6x+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] (x*Sqrt[-(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)
```

Maple [A] time = 0.069, size = 27, normalized size = 1.2

$$\frac{x(3x+4)}{4+6x}\sqrt{-(2+3x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(2+3*x)^2)^(1/2),x)
```

```
[Out] 1/2*x*(3*x+4)*(-(2+3*x)^2)^(1/2)/(2+3*x)
```

Maxima [A] time = 1.73642, size = 41, normalized size = 1.78

$$\frac{1}{2}\sqrt{-9x^2-12x-4}x+\frac{1}{3}\sqrt{-9x^2-12x-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-9*x^2 - 12*x - 4)*x + 1/3*sqrt(-9*x^2 - 12*x - 4)
```

Fricas [C] time = 2.03856, size = 26, normalized size = 1.13

$$\frac{3}{2}ix^2+2ix$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*I*x^2 + 2*I*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-(3x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(3*x + 2)**2), x)
```

Giac [C] time = 1.23801, size = 35, normalized size = 1.52

$$-\frac{1}{2}i\left(3x^2+4x\right)\operatorname{sgn}(-3x-2)-\frac{2}{3}i\operatorname{sgn}(-3x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*I*(3*x^2 + 4*x)*sgn(-3*x - 2) - 2/3*I*sgn(-3*x - 2)
```

3.73 $\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx$

Optimal. Leaf size=29

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-9x^2-12x-4}}$$

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-4 - 12*x - 9*x^2])
```

Rubi [A] time = 0.0056675, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {608, 31}

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-9x^2-12x-4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-4 - 12*x - 9*x^2])
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx &= -\left(-\frac{(-6-9x)\int \frac{1}{-6-9x}\,dx}{\sqrt{-4-12x-9x^2}}\right)\\ &= \frac{(2+3x)\log(2+3x)}{3\sqrt{-4-12x-9x^2}}\end{aligned}$$

Mathematica [A] time = 0.00384, size = 28, normalized size = 0.97

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

Maple [A] time = 0.122, size = 25, normalized size = 0.9

$$\frac{(2+3x)\ln(2+3x)}{3}\frac{1}{\sqrt{-(2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(2+3*x)^2)^(1/2),x)
```

```
[Out] 1/3*(2+3*x)*ln(2+3*x)/(-(2+3*x)^2)^(1/2)
```

Maxima [C] time = 1.69936, size = 8, normalized size = 0.28

$$\frac{1}{3}i\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*I*log(x + 2/3)
```

Fricas [C] time = 2.02109, size = 28, normalized size = 0.97

$$-\frac{1}{3}i\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*I*log(x + 2/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-(3x+2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-(3*x + 2)**2), x)
```

Giac [C] time = 1.3579, size = 31, normalized size = 1.07

$$\frac{i\log\left((-3ix-2i)\operatorname{sgn}(-3x-2)\right)}{3\operatorname{sgn}(-3x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*I*log((-3*I*x - 2*I)*sgn(-3*x - 2))/sgn(-3*x - 2)
```

3.74 $\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 \, dx$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+1)^{11}}{22528c^6}+\frac{(-b-2cx+1)^{10}}{2048c^6}-\frac{5(-b-2cx+1)^9}{2304c^6}+\frac{5(-b-2cx+1)^8}{1024c^6}-\frac{5(-b-2cx+1)^7}{896c^6}+\frac{(-b-2cx+1)^6}{384c^6}$$

```
[Out] (1 - b - 2*c*x)^6/(384*c^6) - (5*(1 - b - 2*c*x)^7)/(896*c^6) + (5*(1 - b -
 2*c*x)^8)/(1024*c^6) - (5*(1 - b - 2*c*x)^9)/(2304*c^6) + (1 - b - 2*c*x)^
10/(2048*c^6) - (1 - b - 2*c*x)^11/(22528*c^6)
```

Rubi [A] time = 0.139885, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {610, 43}

$$-\frac{(-b-2cx+1)^{11}}{22528c^6}+\frac{(-b-2cx+1)^{10}}{2048c^6}-\frac{5(-b-2cx+1)^9}{2304c^6}+\frac{5(-b-2cx+1)^8}{1024c^6}-\frac{5(-b-2cx+1)^7}{896c^6}+\frac{(-b-2cx+1)^6}{384c^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (1 - b - 2*c*x)^6/(384*c^6) - (5*(1 - b - 2*c*x)^7)/(896*c^6) + (5*(1 - b -
 2*c*x)^8)/(1024*c^6) - (5*(1 - b - 2*c*x)^9)/(2304*c^6) + (1 - b - 2*c*x)^
10/(2048*c^6) - (1 - b - 2*c*x)^11/(22528*c^6)
```

Rule 610

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 \, dx &= \frac{\int \left(\frac{1}{2}(-1+b)+cx\right)^5 \left(\frac{1+b}{2}+cx\right)^5 \, dx}{c^5}\\
&= \frac{\int \left(\left(\frac{1}{2}(-1+b)+cx\right)^5+5\left(\frac{1}{2}(-1+b)+cx\right)^6+10\left(\frac{1}{2}(-1+b)+cx\right)^7+10\left(\frac{1}{2}(-1+b)+\right.\right.}{c^5}\\
&= \frac{(1-b-2cx)^6}{384c^6}-\frac{5(1-b-2cx)^7}{896c^6}+\frac{5(1-b-2cx)^8}{1024c^6}-\frac{5(1-b-2cx)^9}{2304c^6}+\frac{(1-b-2cx)^{10}}{2048c^6}
\end{aligned}$$

Mathematica [A] time = 0.0338732, size = 207, normalized size = 1.9

$$\frac{5}{36}\left(9b^2-1\right)c^3x^9+\frac{5}{8}\left(3b^3-b\right)c^2x^8+\frac{5b\left(b^2-1\right)^2\left(3b^2-1\right)x^4}{64c^2}+\frac{5\left(b^2-1\right)^3\left(9b^2-1\right)x^3}{768c^3}+\frac{5b\left(b^2-1\right)^4x^2}{512c^4}+\frac{\left(b^2-1\right.}{1024c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-1 + b^2)^5*x)/(1024*c^5) + (5*b*(-1 + b^2)^4*x^2)/(512*c^4) + (5*(-1 + b
^2)^3*(-1 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-1 + b^2)^2*(-1 + 3*b^2)*x^4)/(64
*c^2) + ((-1 + b^2)*(1 - 14*b^2 + 21*b^4)*x^5)/(32*c) + (b*(15 - 70*b^2 + 6
3*b^4)*x^6)/48 + (5*(1 - 14*b^2 + 21*b^4)*c*x^7)/56 + (5*(-b + 3*b^3)*c^2*x
^8)/8 + (5*(-1 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

Maple [B] time = 0.079, size = 636, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*(b^2-1)/c+b*x+c*x^2)^5,x)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-1)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-1
/2)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-1)*c^2*b+b*(2*(3/2*b^2-1/2)*c^2+4*b^2*c^2
)+c*((b^2-1)*c*b+4*(3/2*b^2-1/2)*b*c))*x^8+1/7*(1/4*(b^2-1)/c*(2*(3/2*b^2-1
/2)*c^2+4*b^2*c^2)+b*((b^2-1)*c*b+4*(3/2*b^2-1/2)*b*c)+c*(1/8*(b^2-1)^2+2*(
b^2-1)*b^2+(3/2*b^2-1/2)^2))*x^7+1/6*(1/4*(b^2-1)/c*((b^2-1)*c*b+4*(3/2*b^2
-1/2)*b*c)+b*(1/8*(b^2-1)^2+2*(b^2-1)*b^2+(3/2*b^2-1/2)^2)+c*(1/4*(b^2-1)^2
/c*b+(b^2-1)/c*b*(3/2*b^2-1/2)))*x^6+1/5*(1/4*(b^2-1)/c*(1/8*(b^2-1)^2+2*(b
^2-1)*b^2+(3/2*b^2-1/2)^2)+b*(1/4*(b^2-1)^2/c*b+(b^2-1)/c*b*(3/2*b^2-1/2))+
c*(1/8*(b^2-1)^2/c^2*(3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2*b^2))*x^5+1/4*(1/4*(b^
2-1)/c*(1/4*(b^2-1)^2/c*b+(b^2-1)/c*b*(3/2*b^2-1/2))+b*(1/8*(b^2-1)^2/c^2*(
3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2*b^2)+1/16/c^2*(b^2-1)^3*b)*x^4+1/3*(1/4*(b^2
-1)/c*(1/8*(b^2-1)^2/c^2*(3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2*b^2)+1/16*b^2*(b^2
-1)^3/c^3+1/256/c^3*(b^2-1)^4)*x^3+5/512*(b^2-1)^4/c^4*b*x^2+1/1024*(b^2-1)
^5/c^5*x
```

Maxima [B] time = 1.12539, size = 316, normalized size = 2.9

$$\frac{1}{11}c^5x^{11}+\frac{1}{2}bc^4x^{10}+\frac{10}{9}b^2c^3x^9+\frac{5}{4}b^3c^2x^8+\frac{5}{7}b^4cx^7+\frac{1}{6}b^5x^6+\frac{5\left(2\,cx^3+3\,bx^2\right)\left(b^2-1\right)^4}{1536\,c^4}+\frac{\left(6\,c^2x^5+15\,bcx^4+10\,b^2x\right.}{192\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b
^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 1)^4/c^4 + 1/192
*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 1)^3/c^3 + 1/224*(20*c^3*x^7
+ 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 1)^2/c^2 + 1/504*(70*c^4
*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2
- 1)/c + 1/1024*(b^2 - 1)^5*x/c^5
```

Fricas [B] time = 2.10253, size = 585, normalized size = 5.37

$$\frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+98560\left(9\,b^2-1\right)c^8x^9+443520\left(3\,b^3-b\right)c^7x^8+63360\left(21\,b^4-14\,b^2+1\right)c^6x^7+14784\,(6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 1)*c^8*x^9 +
 443520*(3*b^3 - b)*c^7*x^8 + 63360*(21*b^4 - 14*b^2 + 1)*c^6*x^7 + 14784*(
63*b^5 - 70*b^3 + 15*b)*c^5*x^6 + 22176*(21*b^6 - 35*b^4 + 15*b^2 - 1)*c^4*
x^5 + 55440*(3*b^7 - 7*b^5 + 5*b^3 - b)*c^3*x^4 + 4620*(9*b^8 - 28*b^6 + 30
*b^4 - 12*b^2 + 1)*c^2*x^3 + 6930*(b^9 - 4*b^7 + 6*b^5 - 4*b^3 + b)*c*x^2 +
 693*(b^10 - 5*b^8 + 10*b^6 - 10*b^4 + 5*b^2 - 1)*x)/c^5
```

Sympy [B] time = 0.209606, size = 253, normalized size = 2.32

$$\frac{bc^4x^{10}}{2} + \frac{c^5x^{11}}{11} + x^9\left(\frac{5b^2c^3}{4} - \frac{5c^3}{36}\right) + x^8\left(\frac{15b^3c^2}{8} - \frac{5bc^2}{8}\right) + x^7\left(\frac{15b^4c}{8} - \frac{5b^2c}{4} + \frac{5c}{56}\right) + x^6\left(\frac{21b^5}{16} - \frac{35b^3}{24} + \frac{5b}{16}\right) + \frac{x^5(\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b**2-1)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/36) + x**8*(1
5*b**3*c**2/8 - 5*b*c**2/8) + x**7*(15*b**4*c/8 - 5*b**2*c/4 + 5*c/56) + x*
*6*(21*b**5/16 - 35*b**3/24 + 5*b/16) + x**5*(21*b**6 - 35*b**4 + 15*b**2 -
 1)/(32*c) + x**4*(15*b**7 - 35*b**5 + 25*b**3 - 5*b)/(64*c**2) + x**3*(45*
b**8 - 140*b**6 + 150*b**4 - 60*b**2 + 5)/(768*c**3) + x**2*(5*b**9 - 20*b*
*7 + 30*b**5 - 20*b**3 + 5*b)/(512*c**4) + x*(b**10 - 5*b**8 + 10*b**6 - 10
*b**4 + 5*b**2 - 1)/(1024*c**5)
```

Giac [B] time = 1.33613, size = 451, normalized size = 4.14

$$\frac{64512\,c^{10}x^{11} + 354816\,bc^9x^{10} + 887040\,b^2c^8x^9 + 1330560\,b^3c^7x^8 + 1330560\,b^4c^6x^7 - 98560\,c^8x^9 + 931392\,b^5c^5x^6 - 4\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 133056
0*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 98560*c^8*x^9 + 931392*b^5*c^5*x^6 -
443520*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 887040*b^2*c^6*x^7 + 166320*b^7*c^3
*x^4 - 1034880*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 776160*b^4*c^4*x^5 + 63360
*c^6*x^7 + 6930*b^9*c*x^2 - 388080*b^5*c^3*x^4 + 221760*b*c^5*x^6 + 693*b^1
0*x - 129360*b^6*c^2*x^3 + 332640*b^2*c^4*x^5 - 27720*b^7*c*x^2 + 277200*b^
3*c^3*x^4 - 3465*b^8*x + 138600*b^4*c^2*x^3 - 22176*c^4*x^5 + 41580*b^5*c*x
^2 - 55440*b*c^3*x^4 + 6930*b^6*x - 55440*b^2*c^2*x^3 - 27720*b^3*c*x^2 - 6
930*b^4*x + 4620*c^2*x^3 + 6930*b*c*x^2 + 3465*b^2*x - 693*x)/c^5
```

3.75 $\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+2)^{11}}{22528c^6}+\frac{(-b-2cx+2)^{10}}{1024c^6}-\frac{5(-b-2cx+2)^9}{576c^6}+\frac{5(-b-2cx+2)^8}{128c^6}-\frac{5(-b-2cx+2)^7}{56c^6}+\frac{(-b-2cx+2)^6}{12c^6}$$

```
[Out] (2 - b - 2*c*x)^6/(12*c^6) - (5*(2 - b - 2*c*x)^7)/(56*c^6) + (5*(2 - b - 2
*c*x)^8)/(128*c^6) - (5*(2 - b - 2*c*x)^9)/(576*c^6) + (2 - b - 2*c*x)^10/(
1024*c^6) - (2 - b - 2*c*x)^11/(22528*c^6)
```

Rubi [A] time = 0.136561, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {610, 43}

$$-\frac{(-b-2cx+2)^{11}}{22528c^6}+\frac{(-b-2cx+2)^{10}}{1024c^6}-\frac{5(-b-2cx+2)^9}{576c^6}+\frac{5(-b-2cx+2)^8}{128c^6}-\frac{5(-b-2cx+2)^7}{56c^6}+\frac{(-b-2cx+2)^6}{12c^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (2 - b - 2*c*x)^6/(12*c^6) - (5*(2 - b - 2*c*x)^7)/(56*c^6) + (5*(2 - b - 2
*c*x)^8)/(128*c^6) - (5*(2 - b - 2*c*x)^9)/(576*c^6) + (2 - b - 2*c*x)^10/(
1024*c^6) - (2 - b - 2*c*x)^11/(22528*c^6)
```

Rule 610

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{\int \left(\frac{1}{2}(-2+b)+cx\right)^5 \left(\frac{2+b}{2}+cx\right)^5 dx}{c^5}$$
$$= \frac{\int \left(32\left(\frac{1}{2}(-2+b)+cx\right)^5+80\left(\frac{1}{2}(-2+b)+cx\right)^6+80\left(\frac{1}{2}(-2+b)+cx\right)^7+40\left(\frac{1}{2}(-2+b)\right.\right.}{c^5}$$
$$= \frac{(2-b-2cx)^6}{12c^6}-\frac{5(2-b-2cx)^7}{56c^6}+\frac{5(2-b-2cx)^8}{128c^6}-\frac{5(2-b-2cx)^9}{576c^6}+\frac{(2-b-2cx)^{10}}{1024c^6}-$$

Mathematica [A] time = 0.0452142, size = 207, normalized size = 1.9

$$\frac{5}{36}\left(9b^2-4\right)c^3x^9+\frac{5}{8}\left(3b^3-4b\right)c^2x^8+\frac{5b\left(b^2-4\right)^2\left(3b^2-4\right)x^4}{64c^2}+\frac{5\left(b^2-4\right)^3\left(9b^2-4\right)x^3}{768c^3}+\frac{5b\left(b^2-4\right)^4x^2}{512c^4}+\frac{\left(b^2-4\right)^5}{1024c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-4 + b^2)^5*x)/(1024*c^5) + (5*b*(-4 + b^2)^4*x^2)/(512*c^4) + (5*(-4 + b
^2)^3*(-4 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-4 + b^2)^2*(-4 + 3*b^2)*x^4)/(64
*c^2) + ((-4 + b^2)*(16 - 56*b^2 + 21*b^4)*x^5)/(32*c) + (b*(240 - 280*b^2
+ 63*b^4)*x^6)/48 + (5*(16 - 56*b^2 + 21*b^4)*c*x^7)/56 + (5*(-4*b + 3*b^3)
*c^2*x^8)/8 + (5*(-4 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

Maple [B] time = 0.073, size = 636, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*(b^2-4)/c+b*x+c*x^2)^5,x)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-4)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-2
)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-4)*c^2*b+b*(2*(3/2*b^2-2)*c^2+4*b^2*c^2)+c*
((b^2-4)*c*b+4*(3/2*b^2-2)*b*c))*x^8+1/7*(1/4*(b^2-4)/c*(2*(3/2*b^2-2)*c^2+
4*b^2*c^2)+b*((b^2-4)*c*b+4*(3/2*b^2-2)*b*c)+c*(1/8*(b^2-4)^2+2*(b^2-4)*b^2
+(3/2*b^2-2)^2))*x^7+1/6*(1/4*(b^2-4)/c*((b^2-4)*c*b+4*(3/2*b^2-2)*b*c)+b*(
1/8*(b^2-4)^2+2*(b^2-4)*b^2+(3/2*b^2-2)^2)+c*(1/4*(b^2-4)^2/c*b+(b^2-4)/c*b
*(3/2*b^2-2)))*x^6+1/5*(1/4*(b^2-4)/c*(1/8*(b^2-4)^2+2*(b^2-4)*b^2+(3/2*b^2
-2)^2)+b*(1/4*(b^2-4)^2/c*b+(b^2-4)/c*b*(3/2*b^2-2))+c*(1/8*(b^2-4)^2/c^2*(
3/2*b^2-2)+1/4*(b^2-4)^2/c^2*b^2))*x^5+1/4*(1/4*(b^2-4)/c*(1/4*(b^2-4)^2/c*
b+(b^2-4)/c*b*(3/2*b^2-2))+b*(1/8*(b^2-4)^2/c^2*(3/2*b^2-2)+1/4*(b^2-4)^2/c
^2*b^2)+1/16/c^2*(b^2-4)^3*b)*x^4+1/3*(1/4*(b^2-4)/c*(1/8*(b^2-4)^2/c^2*(3/
2*b^2-2)+1/4*(b^2-4)^2/c^2*b^2)+1/16*b^2*(b^2-4)^3/c^3+1/256/c^3*(b^2-4)^4)
*x^3+5/512*(b^2-4)^4/c^4*b*x^2+1/1024*(b^2-4)^5/c^5*x
```

Maxima [B] time = 1.07737, size = 316, normalized size = 2.9

$$\frac{1}{11}c^5x^{11} + \frac{1}{2}bc^4x^{10} + \frac{10}{9}b^2c^3x^9 + \frac{5}{4}b^3c^2x^8 + \frac{5}{7}b^4cx^7 + \frac{1}{6}b^5x^6 + \frac{5\left(2\,cx^3 + 3\,bx^2\right)\left(b^2 - 4\right)^4}{1536\,c^4} + \frac{\left(6\,c^2x^5 + 15\,bcx^4 + 10\right.}{192\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b
^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 4)^4/c^4 + 1/192
*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 4)^3/c^3 + 1/224*(20*c^3*x^7
+ 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 4)^2/c^2 + 1/504*(70*c^4
*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2
- 4)/c + 1/1024*(b^2 - 4)^5*x/c^5
```

Fricas [B] time = 2.13781, size = 632, normalized size = 5.8

$$\frac{64512\,c^{10}x^{11} + 354816\,bc^9x^{10} + 98560\left(9\,b^2 - 4\right)c^8x^9 + 443520\left(3\,b^3 - 4\,b\right)c^7x^8 + 63360\left(21\,b^4 - 56\,b^2 + 16\right)c^6x^7 + 14}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 4)*c^8*x^9 +
 443520*(3*b^3 - 4*b)*c^7*x^8 + 63360*(21*b^4 - 56*b^2 + 16)*c^6*x^7 + 1478
4*(63*b^5 - 280*b^3 + 240*b)*c^5*x^6 + 22176*(21*b^6 - 140*b^4 + 240*b^2 -
64)*c^4*x^5 + 55440*(3*b^7 - 28*b^5 + 80*b^3 - 64*b)*c^3*x^4 + 4620*(9*b^8
- 112*b^6 + 480*b^4 - 768*b^2 + 256)*c^2*x^3 + 6930*(b^9 - 16*b^7 + 96*b^5
- 256*b^3 + 256*b)*c*x^2 + 693*(b^10 - 20*b^8 + 160*b^6 - 640*b^4 + 1280*b^
2 - 1024)*x)/c^5
```

Sympy [B] time = 0.266666, size = 250, normalized size = 2.29

$$\frac{bc^4x^{10}}{2}+\frac{c^5x^{11}}{11}+x^9\left(\frac{5b^2c^3}{4}-\frac{5c^3}{9}\right)+x^8\left(\frac{15b^3c^2}{8}-\frac{5bc^2}{2}\right)+x^7\left(\frac{15b^4c}{8}-5b^2c+\frac{10c}{7}\right)+x^6\left(\frac{21b^5}{16}-\frac{35b^3}{6}+5b\right)+\frac{x^5\left(21b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b**2-4)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/9) + x**8*(15
*b**3*c**2/8 - 5*b*c**2/2) + x**7*(15*b**4*c/8 - 5*b**2*c + 10*c/7) + x**6*
(21*b**5/16 - 35*b**3/6 + 5*b) + x**5*(21*b**6 - 140*b**4 + 240*b**2 - 64)/
(32*c) + x**4*(15*b**7 - 140*b**5 + 400*b**3 - 320*b)/(64*c**2) + x**3*(45*
b**8 - 560*b**6 + 2400*b**4 - 3840*b**2 + 1280)/(768*c**3) + x**2*(5*b**9 -
 80*b**7 + 480*b**5 - 1280*b**3 + 1280*b)/(512*c**4) + x*(b**10 - 20*b**8 +
 160*b**6 - 640*b**4 + 1280*b**2 - 1024)/(1024*c**5)
```

Giac [B] time = 1.28737, size = 451, normalized size = 4.14

$$\frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+887040\,b^2c^8x^9+1330560\,b^3c^7x^8+1330560\,b^4c^6x^7-394240\,c^8x^9+931392\,b^5c^5x^6-177}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 133056
0*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 394240*c^8*x^9 + 931392*b^5*c^5*x^6 -
 1774080*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 3548160*b^2*c^6*x^7 + 166320*b^7*
c^3*x^4 - 4139520*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 3104640*b^4*c^4*x^5 + 1
013760*c^6*x^7 + 6930*b^9*c*x^2 - 1552320*b^5*c^3*x^4 + 3548160*b*c^5*x^6 +
 693*b^10*x - 517440*b^6*c^2*x^3 + 5322240*b^2*c^4*x^5 - 110880*b^7*c*x^2 +
 4435200*b^3*c^3*x^4 - 13860*b^8*x + 2217600*b^4*c^2*x^3 - 1419264*c^4*x^5
+ 665280*b^5*c*x^2 - 3548160*b*c^3*x^4 + 110880*b^6*x - 3548160*b^2*c^2*x^3
 - 1774080*b^3*c*x^2 - 443520*b^4*x + 1182720*c^2*x^3 + 1774080*b*c*x^2 + 8
87040*b^2*x - 709632*x)/c^5
```

3.76 $\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+3)^{11}}{22528c^6}+\frac{3(-b-2cx+3)^{10}}{2048c^6}-\frac{5(-b-2cx+3)^9}{256c^6}+\frac{135(-b-2cx+3)^8}{1024c^6}-\frac{405(-b-2cx+3)^7}{896c^6}+\frac{81(-b-2c}{128c}$$

```
[Out] (81*(3 - b - 2*c*x)^6)/(128*c^6) - (405*(3 - b - 2*c*x)^7)/(896*c^6) + (135
*(3 - b - 2*c*x)^8)/(1024*c^6) - (5*(3 - b - 2*c*x)^9)/(256*c^6) + (3*(3 -
b - 2*c*x)^10)/(2048*c^6) - (3 - b - 2*c*x)^11/(22528*c^6)
```

Rubi [A] time = 0.142473, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {610, 43}

$$-\frac{(-b-2cx+3)^{11}}{22528c^6}+\frac{3(-b-2cx+3)^{10}}{2048c^6}-\frac{5(-b-2cx+3)^9}{256c^6}+\frac{135(-b-2cx+3)^8}{1024c^6}-\frac{405(-b-2cx+3)^7}{896c^6}+\frac{81(-b-2c}{128c}$$

Antiderivative was successfully verified.

```
[In] Int[((-9 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (81*(3 - b - 2*c*x)^6)/(128*c^6) - (405*(3 - b - 2*c*x)^7)/(896*c^6) + (135
*(3 - b - 2*c*x)^8)/(1024*c^6) - (5*(3 - b - 2*c*x)^9)/(256*c^6) + (3*(3 -
b - 2*c*x)^10)/(2048*c^6) - (3 - b - 2*c*x)^11/(22528*c^6)
```

Rule 610

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx &= \frac{\int \left(\frac{1}{2}(-3+b)+cx\right)^5 \left(\frac{3+b}{2}+cx\right)^5 dx}{c^5}\\ &= \frac{\int \left(243\left(\frac{1}{2}(-3+b)+cx\right)^5+405\left(\frac{1}{2}(-3+b)+cx\right)^6+270\left(\frac{1}{2}(-3+b)+cx\right)^7+90\left(\frac{1}{2}(-\right.\right.}{c^5}\\ &= \frac{81(3-b-2cx)^6}{128c^6}-\frac{405(3-b-2cx)^7}{896c^6}+\frac{135(3-b-2cx)^8}{1024c^6}-\frac{5(3-b-2cx)^9}{256c^6}+\frac{3(3-b}{20}\end{aligned}$$

Mathematica [A] time = 0.0312584, size = 199, normalized size = 1.83

$$\frac{5}{4}\left(b^2-1\right)c^3x^9+\frac{15}{8}\left(b^3-3b\right)c^2x^8+\frac{15b\left(b^2-9\right)^2\left(b^2-3\right)x^4}{64c^2}+\frac{15\left(b^2-9\right)^3\left(b^2-1\right)x^3}{256c^3}+\frac{5b\left(b^2-9\right)^4x^2}{512c^4}+\frac{\left(b^2-9\right)^5}{1024c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-9 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-9 + b^2)^5*x)/(1024*c^5) + (5*b*(-9 + b^2)^4*x^2)/(512*c^4) + (15*(-9 +
b^2)^3*(-1 + b^2)*x^3)/(256*c^3) + (15*b*(-9 + b^2)^2*(-3 + b^2)*x^4)/(64*c
^2) + (3*(-9 + b^2)*(27 - 42*b^2 + 7*b^4)*x^5)/(32*c) + (3*b*(135 - 70*b^2
+ 7*b^4)*x^6)/16 + (15*(27 - 42*b^2 + 7*b^4)*c*x^7)/56 + (15*(-3*b + b^3)*c
^2*x^8)/8 + (5*(-1 + b^2)*c^3*x^9)/4 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

Maple [B] time = 0.072, size = 636, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*(b^2-9)/c+b*x+c*x^2)^5,x)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-9)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-9
/2)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-9)*c^2*b+b*(2*(3/2*b^2-9/2)*c^2+4*b^2*c^2
)+c*((b^2-9)*c*b+4*(3/2*b^2-9/2)*b*c))*x^8+1/7*(1/4*(b^2-9)/c*(2*(3/2*b^2-9
/2)*c^2+4*b^2*c^2)+b*((b^2-9)*c*b+4*(3/2*b^2-9/2)*b*c)+c*(1/8*(b^2-9)^2+2*(
b^2-9)*b^2+(3/2*b^2-9/2)^2))*x^7+1/6*(1/4*(b^2-9)/c*((b^2-9)*c*b+4*(3/2*b^2
-9/2)*b*c)+b*(1/8*(b^2-9)^2+2*(b^2-9)*b^2+(3/2*b^2-9/2)^2)+c*(1/4*(b^2-9)^2
/c*b+(b^2-9)/c*b*(3/2*b^2-9/2)))*x^6+1/5*(1/4*(b^2-9)/c*(1/8*(b^2-9)^2+2*(b
^2-9)*b^2+(3/2*b^2-9/2)^2)+b*(1/4*(b^2-9)^2/c*b+(b^2-9)/c*b*(3/2*b^2-9/2))+
c*(1/8*(b^2-9)^2/c^2*(3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2*b^2))*x^5+1/4*(1/4*(b^
2-9)/c*(1/4*(b^2-9)^2/c*b+(b^2-9)/c*b*(3/2*b^2-9/2))+b*(1/8*(b^2-9)^2/c^2*(
3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2*b^2)+1/16/c^2*(b^2-9)^3*b)*x^4+1/3*(1/4*(b^2
-9)/c*(1/8*(b^2-9)^2/c^2*(3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2*b^2)+1/16*b^2*(b^2
-9)^3/c^3+1/256/c^3*(b^2-9)^4)*x^3+5/512*(b^2-9)^4/c^4*b*x^2+1/1024*(b^2-9)
^5/c^5*x
```

Maxima [B] time = 1.14294, size = 316, normalized size = 2.9

$$\frac{1}{11}\,c^5x^{11} + \frac{1}{2}\,bc^4x^{10} + \frac{10}{9}\,b^2c^3x^9 + \frac{5}{4}\,b^3c^2x^8 + \frac{5}{7}\,b^4cx^7 + \frac{1}{6}\,b^5x^6 + \frac{5\left(2\,cx^3 + 3\,bx^2\right)\left(b^2-9\right)^4}{1536\,c^4} + \frac{\left(6\,c^2x^5 + 15\,bcx^4 + 10\,b^2x\right.}{192\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b
^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 9)^4/c^4 + 1/192
*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 9)^3/c^3 + 1/224*(20*c^3*x^7
+ 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 9)^2/c^2 + 1/504*(70*c^4
*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2
- 9)/c + 1/1024*(b^2 - 9)^5*x/c^5
```

Fricas [B] time = 2.20788, size = 618, normalized size = 5.67

$$\frac{7168\,c^{10}x^{11} + 39424\,bc^9x^{10} + 98560\left(b^2-1\right)c^8x^9 + 147840\left(b^3-3\,b\right)c^7x^8 + 21120\left(7\,b^4-42\,b^2+27\right)c^6x^7 + 14784\left(7\,b^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/78848*(7168*c^10*x^11 + 39424*b*c^9*x^10 + 98560*(b^2 - 1)*c^8*x^9 + 1478
40*(b^3 - 3*b)*c^7*x^8 + 21120*(7*b^4 - 42*b^2 + 27)*c^6*x^7 + 14784*(7*b^5
 - 70*b^3 + 135*b)*c^5*x^6 + 7392*(7*b^6 - 105*b^4 + 405*b^2 - 243)*c^4*x^5
 + 18480*(b^7 - 21*b^5 + 135*b^3 - 243*b)*c^3*x^4 + 4620*(b^8 - 28*b^6 + 27
0*b^4 - 972*b^2 + 729)*c^2*x^3 + 770*(b^9 - 36*b^7 + 486*b^5 - 2916*b^3 + 6
561*b)*c*x^2 + 77*(b^10 - 45*b^8 + 810*b^6 - 7290*b^4 + 32805*b^2 - 59049)*
x)/c^5
```

Sympy [B] time = 0.292203, size = 253, normalized size = 2.32

$$\frac{bc^4x^{10}}{2} + \frac{c^5x^{11}}{11} + x^9\left(\frac{5b^2c^3}{4} - \frac{5c^3}{4}\right) + x^8\left(\frac{15b^3c^2}{8} - \frac{45bc^2}{8}\right) + x^7\left(\frac{15b^4c}{8} - \frac{45b^2c}{4} + \frac{405c}{56}\right) + x^6\left(\frac{21b^5}{16} - \frac{105b^3}{8} + \frac{405b}{16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b**2-9)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/4) + x**8*(15
*b**3*c**2/8 - 45*b*c**2/8) + x**7*(15*b**4*c/8 - 45*b**2*c/4 + 405*c/56) +
 x**6*(21*b**5/16 - 105*b**3/8 + 405*b/16) + x**5*(21*b**6 - 315*b**4 + 121
5*b**2 - 729)/(32*c) + x**4*(15*b**7 - 315*b**5 + 2025*b**3 - 3645*b)/(64*c
**2) + x**3*(15*b**8 - 420*b**6 + 4050*b**4 - 14580*b**2 + 10935)/(256*c**3
) + x**2*(5*b**9 - 180*b**7 + 2430*b**5 - 14580*b**3 + 32805*b)/(512*c**4)
+ x*(b**10 - 45*b**8 + 810*b**6 - 7290*b**4 + 32805*b**2 - 59049)/(1024*c**
5)
```

Giac [B] time = 1.37238, size = 451, normalized size = 4.14

$$\frac{7168\,c^{10}x^{11} + 39424\,bc^9x^{10} + 98560\,b^2c^8x^9 + 147840\,b^3c^7x^8 + 147840\,b^4c^6x^7 - 98560\,c^8x^9 + 103488\,b^5c^5x^6 - 443520}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

```
[Out] 1/78848*(7168*c^10*x^11 + 39424*b*c^9*x^10 + 98560*b^2*c^8*x^9 + 147840*b^3
*c^7*x^8 + 147840*b^4*c^6*x^7 - 98560*c^8*x^9 + 103488*b^5*c^5*x^6 - 443520
*b*c^7*x^8 + 51744*b^6*c^4*x^5 - 887040*b^2*c^6*x^7 + 18480*b^7*c^3*x^4 - 1
034880*b^3*c^5*x^6 + 4620*b^8*c^2*x^3 - 776160*b^4*c^4*x^5 + 570240*c^6*x^7
 + 770*b^9*c*x^2 - 388080*b^5*c^3*x^4 + 1995840*b*c^5*x^6 + 77*b^10*x - 129
360*b^6*c^2*x^3 + 2993760*b^2*c^4*x^5 - 27720*b^7*c*x^2 + 2494800*b^3*c^3*x
^4 - 3465*b^8*x + 1247400*b^4*c^2*x^3 - 1796256*c^4*x^5 + 374220*b^5*c*x^2
- 4490640*b*c^3*x^4 + 62370*b^6*x - 4490640*b^2*c^2*x^3 - 2245320*b^3*c*x^2
 - 561330*b^4*x + 3367980*c^2*x^3 + 5051970*b*c*x^2 + 2525985*b^2*x - 45467
73*x)/c^5
```

3.77 $\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+4)^{11}}{22528c^6}+\frac{(-b-2cx+4)^{10}}{512c^6}-\frac{5(-b-2cx+4)^9}{144c^6}+\frac{5(-b-2cx+4)^8}{16c^6}-\frac{10(-b-2cx+4)^7}{7c^6}+\frac{8(-b-2cx+4)^6}{3c^6}$$

```
[Out] (8*(4 - b - 2*c*x)^6)/(3*c^6) - (10*(4 - b - 2*c*x)^7)/(7*c^6) + (5*(4 - b
- 2*c*x)^8)/(16*c^6) - (5*(4 - b - 2*c*x)^9)/(144*c^6) + (4 - b - 2*c*x)^10
/(512*c^6) - (4 - b - 2*c*x)^11/(22528*c^6)
```

Rubi [A] time = 0.139849, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {610, 43}

$$-\frac{(-b-2cx+4)^{11}}{22528c^6}+\frac{(-b-2cx+4)^{10}}{512c^6}-\frac{5(-b-2cx+4)^9}{144c^6}+\frac{5(-b-2cx+4)^8}{16c^6}-\frac{10(-b-2cx+4)^7}{7c^6}+\frac{8(-b-2cx+4)^6}{3c^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-16 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (8*(4 - b - 2*c*x)^6)/(3*c^6) - (10*(4 - b - 2*c*x)^7)/(7*c^6) + (5*(4 - b
- 2*c*x)^8)/(16*c^6) - (5*(4 - b - 2*c*x)^9)/(144*c^6) + (4 - b - 2*c*x)^10
/(512*c^6) - (4 - b - 2*c*x)^11/(22528*c^6)
```

Rule 610

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{\int \left(\frac{1}{2}(-4+b)+cx\right)^5 \left(\frac{4+b}{2}+cx\right)^5 dx}{c^5}$$
$$= \frac{\int \left(1024\left(\frac{1}{2}(-4+b)+cx\right)^5+1280\left(\frac{1}{2}(-4+b)+cx\right)^6+640\left(\frac{1}{2}(-4+b)+cx\right)^7+160\left(\frac{1}{2}(\right.\right.}{c^5}$$
$$= \frac{8(4-b-2cx)^6}{3c^6}-\frac{10(4-b-2cx)^7}{7c^6}+\frac{5(4-b-2cx)^8}{16c^6}-\frac{5(4-b-2cx)^9}{144c^6}+\frac{(4-b-2cx)^{1}}{512c^6}$$

Mathematica [A] time = 0.0442095, size = 207, normalized size = 1.9

$$\frac{5}{36}\left(9b^2-16\right)c^3x^9+\frac{5}{8}\left(3b^3-16b\right)c^2x^8+\frac{5b\left(b^2-16\right)^2\left(3b^2-16\right)x^4}{64c^2}+\frac{5\left(b^2-16\right)^3\left(9b^2-16\right)x^3}{768c^3}+\frac{5b\left(b^2-16\right)^4x^2}{512c^4}+\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((-16 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-16 + b^2)^5*x)/(1024*c^5) + (5*b*(-16 + b^2)^4*x^2)/(512*c^4) + (5*(-16
+ b^2)^3*(-16 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-16 + b^2)^2*(-16 + 3*b^2)*x^
4)/(64*c^2) + ((-16 + b^2)*(256 - 224*b^2 + 21*b^4)*x^5)/(32*c) + (b*(3840
- 1120*b^2 + 63*b^4)*x^6)/48 + (5*(256 - 224*b^2 + 21*b^4)*c*x^7)/56 + (5*(
-16*b + 3*b^3)*c^2*x^8)/8 + (5*(-16 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 +
 (c^5*x^11)/11
```

Maple [B] time = 0.069, size = 636, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*(b^2-16)/c+b*x+c*x^2)^5,x)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-16)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-
8)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-16)*c^2*b+b*(2*(3/2*b^2-8)*c^2+4*b^2*c^2)+
c*((b^2-16)*c*b+4*(3/2*b^2-8)*b*c))*x^8+1/7*(1/4*(b^2-16)/c*(2*(3/2*b^2-8)*
c^2+4*b^2*c^2)+b*((b^2-16)*c*b+4*(3/2*b^2-8)*b*c)+c*(1/8*(b^2-16)^2+2*(b^2-
16)*b^2+(3/2*b^2-8)^2))*x^7+1/6*(1/4*(b^2-16)/c*((b^2-16)*c*b+4*(3/2*b^2-8)
*b*c)+b*(1/8*(b^2-16)^2+2*(b^2-16)*b^2+(3/2*b^2-8)^2)+c*(1/4*(b^2-16)^2/c*b
+(b^2-16)/c*b*(3/2*b^2-8)))*x^6+1/5*(1/4*(b^2-16)/c*(1/8*(b^2-16)^2+2*(b^2-
16)*b^2+(3/2*b^2-8)^2)+b*(1/4*(b^2-16)^2/c*b+(b^2-16)/c*b*(3/2*b^2-8))+c*(1
/8*(b^2-16)^2/c^2*(3/2*b^2-8)+1/4*(b^2-16)^2/c^2*b^2))*x^5+1/4*(1/4*(b^2-16
)/c*(1/4*(b^2-16)^2/c*b+(b^2-16)/c*b*(3/2*b^2-8))+b*(1/8*(b^2-16)^2/c^2*(3/
2*b^2-8)+1/4*(b^2-16)^2/c^2*b^2)+1/16/c^2*(b^2-16)^3*b)*x^4+1/3*(1/4*(b^2-1
6)/c*(1/8*(b^2-16)^2/c^2*(3/2*b^2-8)+1/4*(b^2-16)^2/c^2*b^2)+1/16*b^2*(b^2-
16)^3/c^3+1/256/c^3*(b^2-16)^4)*x^3+5/512*(b^2-16)^4/c^4*b*x^2+1/1024*(b^2-
16)^5/c^5*x
```

Maxima [B] time = 1.15325, size = 316, normalized size = 2.9

$$\frac{1}{11}c^5x^{11} + \frac{1}{2}bc^4x^{10} + \frac{10}{9}b^2c^3x^9 + \frac{5}{4}b^3c^2x^8 + \frac{5}{7}b^4cx^7 + \frac{1}{6}b^5x^6 + \frac{5\left(2\,cx^3 + 3\,bx^2\right)\left(b^2 - 16\right)^4}{1536\,c^4} + \frac{\left(6\,c^2x^5 + 15\,bcx^4 + 1\right.}{192\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b
^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 16)^4/c^4 + 1/19
2*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 16)^3/c^3 + 1/224*(20*c^3*x^
7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 16)^2/c^2 + 1/504*(70*
c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b
^2 - 16)/c + 1/1024*(b^2 - 16)^5*x/c^5
```

Fricas [B] time = 2.10453, size = 676, normalized size = 6.2

$$\frac{64512\,c^{10}x^{11} + 354816\,bc^{9}x^{10} + 98560\left(9\,b^{2} - 16\right)c^{8}x^{9} + 443520\left(3\,b^{3} - 16\,b\right)c^{7}x^{8} + 63360\left(21\,b^{4} - 224\,b^{2} + 256\right)c^{6}x^{7} + 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 16)*c^8*x^9
+ 443520*(3*b^3 - 16*b)*c^7*x^8 + 63360*(21*b^4 - 224*b^2 + 256)*c^6*x^7 +
14784*(63*b^5 - 1120*b^3 + 3840*b)*c^5*x^6 + 22176*(21*b^6 - 560*b^4 + 3840
*b^2 - 4096)*c^4*x^5 + 55440*(3*b^7 - 112*b^5 + 1280*b^3 - 4096*b)*c^3*x^4
+ 4620*(9*b^8 - 448*b^6 + 7680*b^4 - 49152*b^2 + 65536)*c^2*x^3 + 6930*(b^9
 - 64*b^7 + 1536*b^5 - 16384*b^3 + 65536*b)*c*x^2 + 693*(b^10 - 80*b^8 + 25
60*b^6 - 40960*b^4 + 327680*b^2 - 1048576)*x)/c^5
```

Sympy [B] time = 0.271363, size = 248, normalized size = 2.28

$$\frac{bc^{4}x^{10}}{2} + \frac{c^{5}x^{11}}{11} + x^{9}\left(\frac{5b^{2}c^{3}}{4} - \frac{20c^{3}}{9}\right) + x^{8}\left(\frac{15b^{3}c^{2}}{8} - 10bc^{2}\right) + x^{7}\left(\frac{15b^{4}c}{8} - 20b^{2}c + \frac{160c}{7}\right) + x^{6}\left(\frac{21b^{5}}{16} - \frac{70b^{3}}{3} + 80b\right) + \frac{x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b**2-16)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 20*c**3/9) + x**8*(1
5*b**3*c**2/8 - 10*b*c**2) + x**7*(15*b**4*c/8 - 20*b**2*c + 160*c/7) + x**
6*(21*b**5/16 - 70*b**3/3 + 80*b) + x**5*(21*b**6 - 560*b**4 + 3840*b**2 -
4096)/(32*c) + x**4*(15*b**7 - 560*b**5 + 6400*b**3 - 20480*b)/(64*c**2) +
x**3*(45*b**8 - 2240*b**6 + 38400*b**4 - 245760*b**2 + 327680)/(768*c**3) +
 x**2*(5*b**9 - 320*b**7 + 7680*b**5 - 81920*b**3 + 327680*b)/(512*c**4) +
x*(b**10 - 80*b**8 + 2560*b**6 - 40960*b**4 + 327680*b**2 - 1048576)/(1024*
c**5)
```

Giac [B] time = 1.25856, size = 451, normalized size = 4.14

$$\frac{64512\,c^{10}x^{11} + 354816\,bc^{9}x^{10} + 887040\,b^{2}c^{8}x^{9} + 1330560\,b^{3}c^{7}x^{8} + 1330560\,b^{4}c^{6}x^{7} - 1576960\,c^{8}x^{9} + 931392\,b^{5}c^{5}x^{6} - 70}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 133056
0*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 1576960*c^8*x^9 + 931392*b^5*c^5*x^6
- 7096320*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 14192640*b^2*c^6*x^7 + 166320*b^
7*c^3*x^4 - 16558080*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 12418560*b^4*c^4*x^5
 + 16220160*c^6*x^7 + 6930*b^9*c*x^2 - 6209280*b^5*c^3*x^4 + 56770560*b*c^5
*x^6 + 693*b^10*x - 2069760*b^6*c^2*x^3 + 85155840*b^2*c^4*x^5 - 443520*b^7
*c*x^2 + 70963200*b^3*c^3*x^4 - 55440*b^8*x + 35481600*b^4*c^2*x^3 - 908328
96*c^4*x^5 + 10644480*b^5*c*x^2 - 227082240*b*c^3*x^4 + 1774080*b^6*x - 227
082240*b^2*c^2*x^3 - 113541120*b^3*c*x^2 - 28385280*b^4*x + 302776320*c^2*x
^3 + 454164480*b*c*x^2 + 227082240*b^2*x - 726663168*x)/c^5
```

3.78 $\int \frac{1}{2+4x+3x^2}\,dx$

Optimal. Leaf size=18

$$\frac{\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]
```

Rubi [A] time = 0.0112172, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 204}

$$\frac{\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x + 3*x^2)^(-1),x]
```

```
[Out] ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2+4x+3x^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 4+6x\right)\right)\\ &= \frac{\tan^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0049882, size = 18, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x + 3*x^2)^(-1),x]
```

```
[Out] ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]
```

Maple [A] time = 0.044, size = 17, normalized size = 0.9

$$\frac{\sqrt{2}}{2}\arctan\left(\frac{(4+6x)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+4*x+2),x)
```

```
[Out] 1/2*2^(1/2)*arctan(1/4*(4+6*x)*2^(1/2))
```

Maxima [A] time = 1.78149, size = 22, normalized size = 1.22

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

Fricas [A] time = 2.13459, size = 58, normalized size = 3.22

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

Sympy [A] time = 0.10425, size = 22, normalized size = 1.22

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{3\sqrt{2}x}{2}+\sqrt{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+4*x+2),x)
```

```
[Out] sqrt(2)*atan(3*sqrt(2)*x/2 + sqrt(2))/2
```

Giac [A] time = 1.24023, size = 22, normalized size = 1.22

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

3.79 $\int \frac{1}{4-2\sqrt{3}x+x^2} \, dx$

Optimal. Leaf size=12

$$-\tan^{-1}\left(\sqrt{3}-x\right)$$

```
[Out] -ArcTan[Sqrt[3] - x]
```

Rubi [A] time = 0.0086108, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {618, 204}

$$-\tan^{-1}\left(\sqrt{3}-x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]

[Out] -ArcTan[Sqrt[3] - x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-2\sqrt{3}x+x^2} \, dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-4-x^2} \, dx, x, -2\sqrt{3}+2x\right)\right)\\ &= -\tan^{-1}\left(\sqrt{3}-x\right)\end{aligned}$$

Mathematica [A] time = 0.0142963, size = 12, normalized size = 1.

$$-\tan^{-1}\left(\sqrt{3}-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]

[Out] -ArcTan[Sqrt[3] - x]
```

Maple [A] time = 0.052, size = 9, normalized size = 0.8

$$\arctan\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+x^2-2*x*3^(1/2)),x)
```

```
[Out] arctan(x-3^(1/2))
```

Maxima [A] time = 1.70056, size = 11, normalized size = 0.92

$$\arctan\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="maxima")
```

```
[Out] arctan(x - sqrt(3))
```

Fricas [A] time = 2.27582, size = 28, normalized size = 2.33

$$\arctan\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="fricas")
```

```
[Out] arctan(x - sqrt(3))
```

Sympy [A] time = 0.247866, size = 7, normalized size = 0.58

$$\operatorname{atan}\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x**2-2*x*3**(1/2)),x)
```

```
[Out] atan(x - sqrt(3))
```

Giac [A] time = 1.30593, size = 11, normalized size = 0.92

$$\arctan\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="giac")
```

```
[Out] arctan(x - sqrt(3))
```

3.80 $\int \frac{1}{2+4x-3x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}$$

```
[Out] -(ArcTanh[(2 - 3*x)/Sqrt[10]]/Sqrt[10])
```

Rubi [A] time = 0.0192329, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 206}

$$-\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x - 3*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(2 - 3*x)/Sqrt[10]]/Sqrt[10])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2+4x-3x^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{40-x^2}\,dx, x, 4-6x\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}\end{aligned}$$

Mathematica [A] time = 0.020412, size = 34, normalized size = 1.79

$$\frac{\log\left(3x+\sqrt{10}-2\right)-\log\left(-3x+\sqrt{10}+2\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x - 3*x^2)^(-1),x]
```

```
[Out] (-Log[2 + Sqrt[10] - 3*x] + Log[-2 + Sqrt[10] + 3*x])/(2*Sqrt[10])
```

Maple [A] time = 0.046, size = 17, normalized size = 0.9

$$\frac{\sqrt{10}}{10}\text{Artanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x+2),x)
```

```
[Out] 1/10*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2))
```

Maxima [A] time = 1.49764, size = 36, normalized size = 1.89

$$-\frac{1}{20}\sqrt{10}\log\left(\frac{3x-\sqrt{10}-2}{3x+\sqrt{10}-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2))
```

Fricas [B] time = 2.19858, size = 109, normalized size = 5.74

$$\frac{1}{20}\sqrt{10}\log\left(\frac{9x^2+2\sqrt{10}(3x-2)-12x+14}{3x^2-4x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(10)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*x + 14)/(3*x^2 - 4*x -
 2))
```

Sympy [A] time = 0.145693, size = 39, normalized size = 2.05

$$\frac{\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{20}-\frac{\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x+2),x)
```

```
[Out] sqrt(10)*log(x - 2/3 + sqrt(10)/3)/20 - sqrt(10)*log(x - sqrt(10)/3 - 2/3)/
20
```

Giac [A] time = 1.23253, size = 42, normalized size = 2.21

$$-\frac{1}{20}\sqrt{10}\log\left(\frac{|6x-2\sqrt{10}-4|}{|6x+2\sqrt{10}-4|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4))
```

3.81 $\int \frac{1}{2+5x+3x^2}\,dx$

Optimal. Leaf size=13

$$\log(3x+2)-\log(x+1)$$

```
[Out] -Log[1 + x] + Log[2 + 3*x]
```

Rubi [A] time = 0.0052461, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {616, 31}

$$\log(3x+2)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + 3*x^2)^(-1),x]
```

```
[Out] -Log[1 + x] + Log[2 + 3*x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2+5x+3x^2}\,dx &= 3\int \frac{1}{2+3x}\,dx-3\int \frac{1}{3+3x}\,dx\\ &= -\log(1+x)+\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0028716, size = 13, normalized size = 1.

$$\log(3x+2)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + 3*x^2)^(-1),x]
```

```
[Out] -Log[1 + x] + Log[2 + 3*x]
```

Maple [A] time = 0.048, size = 14, normalized size = 1.1

$$-\ln(1+x)+\ln(2+3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+5*x+2),x)
```

```
[Out] -ln(1+x)+ln(2+3*x)
```

Maxima [A] time = 1.22863, size = 18, normalized size = 1.38

$$\log(3x+2) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2),x, algorithm="maxima")
```

```
[Out] log(3*x + 2) - log(x + 1)
```

Fricas [A] time = 2.12584, size = 36, normalized size = 2.77

$$\log(3x+2) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2),x, algorithm="fricas")
```

```
[Out] log(3*x + 2) - log(x + 1)
```

Sympy [A] time = 0.118538, size = 10, normalized size = 0.77

$$\log\left(x + \frac{2}{3}\right) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+5*x+2),x)
```

```
[Out] log(x + 2/3) - log(x + 1)
```

Giac [A] time = 1.25871, size = 20, normalized size = 1.54

$$\log(|3x+2|) - \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2),x, algorithm="giac")
```

```
[Out] log(abs(3*x + 2)) - log(abs(x + 1))
```

3.82 $\int \frac{1}{2+5x-3x^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{7}\log(3x+1)-\frac{1}{7}\log(2-x)$$

```
[Out] -Log[2 - x]/7 + Log[1 + 3*x]/7
```

Rubi [A] time = 0.0055975, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {616, 31}

$$\frac{1}{7}\log(3x+1)-\frac{1}{7}\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x - 3*x^2)^(-1),x]

[Out] -Log[2 - x]/7 + Log[1 + 3*x]/7
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2+5x-3x^2}\,dx &= -\left(\frac{3}{7}\int \frac{1}{-1-3x}\,dx\right)+\frac{3}{7}\int \frac{1}{6-3x}\,dx\\ &= -\frac{1}{7}\log(2-x)+\frac{1}{7}\log(1+3x)\end{aligned}$$

Mathematica [A] time = 0.0029986, size = 21, normalized size = 1.

$$\frac{1}{7}\log(3x+1)-\frac{1}{7}\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x - 3*x^2)^(-1),x]

[Out] -Log[2 - x]/7 + Log[1 + 3*x]/7
```

Maple [A] time = 0.049, size = 16, normalized size = 0.8

$$\frac{\ln(1+3x)}{7} - \frac{\ln(-2+x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+5*x+2),x)
```

```
[Out] 1/7*ln(1+3*x)-1/7*ln(-2+x)
```

Maxima [A] time = 1.09327, size = 20, normalized size = 0.95

$$\frac{1}{7}\log(3x+1) - \frac{1}{7}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2),x, algorithm="maxima")
```

```
[Out] 1/7*log(3*x + 1) - 1/7*log(x - 2)
```

Fricas [A] time = 2.14062, size = 47, normalized size = 2.24

$$\frac{1}{7}\log(3x+1) - \frac{1}{7}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2),x, algorithm="fricas")
```

```
[Out] 1/7*log(3*x + 1) - 1/7*log(x - 2)
```

Sympy [A] time = 0.16336, size = 14, normalized size = 0.67

$$-\frac{\log(x-2)}{7} + \frac{\log\left(x+\frac{1}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+5*x+2),x)
```

```
[Out] -log(x - 2)/7 + log(x + 1/3)/7
```

Giac [A] time = 1.20764, size = 23, normalized size = 1.1

$$\frac{1}{7}\log(|3x+1|) - \frac{1}{7}\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2),x, algorithm="giac")
```

```
[Out] 1/7*log(abs(3*x + 1)) - 1/7*log(abs(x - 2))
```

3.83 $\int \frac{1}{3+4x+x^2} \, dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(x+2)$$

```
[Out] -ArcTanh[2 + x]
```

Rubi [B] time = 0.0036109, antiderivative size = 17, normalized size of antiderivative = 2.83, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {616, 31}

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x + x^2)^(-1),x]
```

```
[Out] Log[1 + x]/2 - Log[3 + x]/2
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{3+4x+x^2} \, dx &= \frac{1}{2}\int \frac{1}{1+x} \, dx - \frac{1}{2}\int \frac{1}{3+x} \, dx \\
&= \frac{1}{2}\log(1+x) - \frac{1}{2}\log(3+x)
\end{aligned}$$

Mathematica [B] time = 0.0025821, size = 17, normalized size = 2.83

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x + x^2)^(-1),x]
```

```
[Out] Log[1 + x]/2 - Log[3 + x]/2
```

Maple [B] time = 0.047, size = 14, normalized size = 2.3

$$\frac{\ln(1+x)}{2} - \frac{\ln(3+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4*x+3),x)
```

```
[Out] 1/2*ln(1+x)-1/2*ln(3+x)
```

Maxima [B] time = 1.13718, size = 18, normalized size = 3.

$$-\frac{1}{2}\log(x+3) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+3),x, algorithm="maxima")
```

```
[Out] -1/2*log(x + 3) + 1/2*log(x + 1)
```

Fricas [B] time = 2.09493, size = 46, normalized size = 7.67

$$-\frac{1}{2}\log(x+3) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+3),x, algorithm="fricas")
```

```
[Out] -1/2*log(x + 3) + 1/2*log(x + 1)
```

Sympy [B] time = 0.094988, size = 12, normalized size = 2.

$$\frac{\log(x+1)}{2} - \frac{\log(x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4*x+3),x)
```

```
[Out] log(x + 1)/2 - log(x + 3)/2
```

Giac [B] time = 1.26072, size = 20, normalized size = 3.33

$$-\frac{1}{2}\log(|x+3|) + \frac{1}{2}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+3),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(x + 3)) + 1/2*log(abs(x + 1))
```

3.84 $\int \frac{1}{1+\pi x+2x^2} \, dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

```
[Out] (-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

Rubi [A] time = 0.0186286, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Pi*x + 2*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\pi x+2x^2} \, dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-8+\pi^2-x^2} \, dx, x, \pi+4x\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\pi+4x}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}\end{aligned}$$

Mathematica [A] time = 0.0086114, size = 27, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Pi*x + 2*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

Maple [A] time = 0.045, size = 24, normalized size = 0.9

$$-2\frac{1}{\sqrt{\pi^2-8}}\text{Artanh}\left(\frac{\pi+4x}{\sqrt{\pi^2-8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x+2*x^2+1),x)
```

```
[Out] -2*arctanh((Pi+4*x)/(Pi^2-8)^(1/2))/(Pi^2-8)^(1/2)
```

Maxima [A] time = 1.12243, size = 51, normalized size = 1.89

$$\frac{\log\left(\frac{\pi+4x-\sqrt{\pi^2-8}}{\pi+4x+\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+2*x^2+1),x, algorithm="maxima")
```

```
[Out] log((pi + 4*x - sqrt(pi^2 - 8))/(pi + 4*x + sqrt(pi^2 - 8)))/sqrt(pi^2 - 8)
```

Fricas [B] time = 2.27514, size = 130, normalized size = 4.81

$$\frac{\log\left(\frac{\pi^2+4\pi x+8x^2-(\pi+4x)\sqrt{\pi^2-8}-4}{\pi x+2x^2+1}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+2*x^2+1),x, algorithm="fricas")
```

```
[Out] log((pi^2 + 4*pi*x + 8*x^2 - (pi + 4*x)*sqrt(pi^2 - 8) - 4)/(pi*x + 2*x^2 +
 1))/sqrt(pi^2 - 8)
```

Sympy [B] time = 0.192538, size = 76, normalized size = 2.81

$$\frac{\log\left(x-\frac{\pi^2}{4\sqrt{-8+\pi^2}}+\frac{\pi}{4}+\frac{2}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}-\frac{\log\left(x-\frac{2}{\sqrt{-8+\pi^2}}+\frac{\pi}{4}+\frac{\pi^2}{4\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+2*x**2+1),x)
```

```
[Out] log(x - pi**2/(4*sqrt(-8 + pi**2)) + pi/4 + 2/sqrt(-8 + pi**2))/sqrt(-8 + p
i**2) - log(x - 2/sqrt(-8 + pi**2) + pi/4 + pi**2/(4*sqrt(-8 + pi**2)))/sqr
```

```
t(-8 + pi**2)
```

Giac [A] time = 1.22607, size = 54, normalized size = 2.

$$\frac{\log\left(\frac{\left|\pi+4x-\sqrt{\pi^2-8}\right|}{\left|\pi+4x+\sqrt{\pi^2-8}\right|}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+2*x^2+1),x, algorithm="giac")
```

```
[Out] log(abs(pi + 4*x - sqrt(pi^2 - 8))/abs(pi + 4*x + sqrt(pi^2 - 8)))/sqrt(pi^
2 - 8)
```

3.85 $\int \frac{1}{1+\pi x-2x^2} \, dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

```
[Out] (-2*ArcTanh[(Pi - 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

Rubi [A] time = 0.01862, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Pi*x - 2*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(Pi - 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\pi x-2x^2} \, dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{8+\pi^2-x^2} \, dx, x, \pi-4x\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}\end{aligned}$$

Mathematica [A] time = 0.0082617, size = 29, normalized size = 1.07

$$\frac{2\tanh^{-1}\left(\frac{4x-\pi}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Pi*x - 2*x^2)^(-1),x]
```

```
[Out] (2*ArcTanh[(-Pi + 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

Maple [A] time = 0.044, size = 26, normalized size = 1.

$$2\frac{1}{\sqrt{\pi^2+8}}\mathrm{Artanh}\left(\frac{4x-\pi}{\sqrt{\pi^2+8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x-2*x^2+1),x)
```

```
[Out] 2/(Pi^2+8)^(1/2)*arctanh((4*x-Pi)/(Pi^2+8)^(1/2))
```

Maxima [A] time = 1.14767, size = 53, normalized size = 1.96

$$-\frac{\log\left(\frac{\pi-4x+\sqrt{\pi^2+8}}{\pi-4x-\sqrt{\pi^2+8}}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-2*x^2+1),x, algorithm="maxima")
```

```
[Out] -log((pi - 4*x + sqrt(pi^2 + 8))/(pi - 4*x - sqrt(pi^2 + 8)))/sqrt(pi^2 + 8
)
```

Fricas [B] time = 2.18887, size = 131, normalized size = 4.85

$$\frac{\log\left(-\frac{\pi^2-4\pi x+8x^2-(\pi-4x)\sqrt{\pi^2+8}+4}{\pi x-2x^2+1}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-2*x^2+1),x, algorithm="fricas")
```

```
[Out] log(-(pi^2 - 4*pi*x + 8*x^2 - (pi - 4*x)*sqrt(pi^2 + 8) + 4)/(pi*x - 2*x^2
+ 1))/sqrt(pi^2 + 8)
```

Sympy [B] time = 0.320016, size = 76, normalized size = 2.81

$$-\frac{\log\left(x-\frac{\pi}{4}-\frac{\pi^2}{4\sqrt{8+\pi^2}}-\frac{2}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}+\frac{\log\left(x-\frac{\pi}{4}+\frac{2}{\sqrt{8+\pi^2}}+\frac{\pi^2}{4\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-2*x**2+1),x)
```

```
[Out] -log(x - pi/4 - pi**2/(4*sqrt(8 + pi**2)) - 2/sqrt(8 + pi**2))/sqrt(8 + pi*
*2) + log(x - pi/4 + 2/sqrt(8 + pi**2) + pi**2/(4*sqrt(8 + pi**2)))/sqrt(8
+ pi**2)
```

Giac [A] time = 1.27299, size = 61, normalized size = 2.26

$$-\frac{\log\left(\frac{\left|-\pi+4x-\sqrt{\pi^2+8}\right|}{\left|-\pi+4x+\sqrt{\pi^2+8}\right|}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-2*x^2+1),x, algorithm="giac")
```

```
[Out] -log(abs(-pi + 4*x - sqrt(pi^2 + 8))/abs(-pi + 4*x + sqrt(pi^2 + 8)))/sqrt(
pi^2 + 8)
```

3.86 $\int \frac{1}{1+\pi x+3x^2}\,dx$

Optimal. Leaf size=31

$$\frac{2\tan^{-1}\left(\frac{6x+\pi}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

```
[Out] (2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

Rubi [A] time = 0.0202124, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 204}

$$\frac{2\tan^{-1}\left(\frac{6x+\pi}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Pi*x + 3*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\pi x+3x^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-12+\pi^2-x^2}\,dx, x, \pi+6x\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{\pi+6x}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}\end{aligned}$$

Mathematica [A] time = 0.0098903, size = 31, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{6x+\pi}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Pi*x + 3*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

Maple [A] time = 0.045, size = 28, normalized size = 0.9

$$2\,\frac{1}{\sqrt{-\pi^2+12}}\arctan\left(\frac{\pi+6\,x}{\sqrt{-\pi^2+12}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x+3*x^2+1),x)
```

```
[Out] 2*arctan((Pi+6*x)/(-Pi^2+12)^(1/2))/(-Pi^2+12)^(1/2)
```

Maxima [A] time = 1.10936, size = 36, normalized size = 1.16

$$\frac{2\,\arctan\left(\frac{\pi+6\,x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+3*x^2+1),x, algorithm="maxima")
```

```
[Out] 2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)
```

Fricas [A] time = 2.31212, size = 108, normalized size = 3.48

$$\frac{2\,\sqrt{-\pi^2+12}\arctan\left(\frac{(\pi+6\,x)\sqrt{-\pi^2+12}}{\pi^2-12}\right)}{\pi^2-12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-pi^2 + 12)*arctan((pi + 6*x)*sqrt(-pi^2 + 12)/(pi^2 - 12))/(pi^2 -
12)
```

Sympy [C] time = 0.305933, size = 87, normalized size = 2.81

$$-\frac{i\log\left(x+\frac{\pi}{6}-\frac{2i}{\sqrt{12-\pi^2}}+\frac{i\pi^2}{6\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}+\frac{i\log\left(x+\frac{\pi}{6}-\frac{i\pi^2}{6\sqrt{12-\pi^2}}+\frac{2i}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+3*x**2+1),x)
```

```
[Out] -I*log(x + pi/6 - 2*I/sqrt(12 - pi**2) + I*pi**2/(6*sqrt(12 - pi**2)))/sqrt
(12 - pi**2) + I*log(x + pi/6 - I*pi**2/(6*sqrt(12 - pi**2)) + 2*I/sqrt(12
```

```
- pi**2))/sqrt(12 - pi**2)
```

Giac [A] time = 1.24485, size = 36, normalized size = 1.16

$$\frac{2\arctan\left(\frac{\pi+6x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+3*x^2+1),x, algorithm="giac")
```

```
[Out] 2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)
```

3.87 $\int \frac{1}{1+\pi x-3x^2} dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

```
[Out] (-2*ArcTanh[(Pi - 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

Rubi [A] time = 0.0182718, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Pi*x - 3*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(Pi - 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\pi x-3x^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{12+\pi^2-x^2}\,dx, x, \pi-6x\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}\end{aligned}$$

Mathematica [A] time = 0.0083681, size = 29, normalized size = 1.07

$$\frac{2\tanh^{-1}\left(\frac{6x-\pi}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Pi*x - 3*x^2)^(-1),x]
```

```
[Out] (2*ArcTanh[(-Pi + 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

Maple [A] time = 0.054, size = 26, normalized size = 1.

$$2\frac{1}{\sqrt{\pi^2+12}}\text{Artanh}\left(\frac{6x-\pi}{\sqrt{\pi^2+12}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x-3*x^2+1),x)
```

```
[Out] 2/(Pi^2+12)^(1/2)*arctanh((6*x-Pi)/(Pi^2+12)^(1/2))
```

Maxima [A] time = 1.21781, size = 53, normalized size = 1.96

$$-\frac{\log\left(\frac{\pi-6x+\sqrt{\pi^2+12}}{\pi-6x-\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -log((pi - 6*x + sqrt(pi^2 + 12))/(pi - 6*x - sqrt(pi^2 + 12)))/sqrt(pi^2 +
 12)
```

Fricas [B] time = 2.35865, size = 135, normalized size = 5.

$$\frac{\log\left(-\frac{\pi^2-6\pi x+18x^2-(\pi-6x)\sqrt{\pi^2+12}+6}{\pi x-3x^2+1}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-3*x^2+1),x, algorithm="fricas")
```

```
[Out] log(-(pi^2 - 6*pi*x + 18*x^2 - (pi - 6*x)*sqrt(pi^2 + 12) + 6)/(pi*x - 3*x^
2 + 1))/sqrt(pi^2 + 12)
```

Sympy [B] time = 0.291715, size = 76, normalized size = 2.81

$$\frac{\log\left(x-\frac{\pi}{6}+\frac{\pi^2}{6\sqrt{\pi^2+12}}+\frac{2}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}-\frac{\log\left(x-\frac{\pi}{6}-\frac{2}{\sqrt{\pi^2+12}}-\frac{\pi^2}{6\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-3*x**2+1),x)
```

```
[Out] log(x - pi/6 + pi**2/(6*sqrt(pi**2 + 12)) + 2/sqrt(pi**2 + 12))/sqrt(pi**2
+ 12) - log(x - pi/6 - 2/sqrt(pi**2 + 12) - pi**2/(6*sqrt(pi**2 + 12)))/sqr
t(pi**2 + 12)
```

Giac [A] time = 1.31224, size = 61, normalized size = 2.26

$$-\frac{\log\left(\frac{\left|-\pi+6x-\sqrt{\pi^2+12}\right|}{\left|-\pi+6x+\sqrt{\pi^2+12}\right|}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-3*x^2+1),x, algorithm="giac")
```

```
[Out] -log(abs(-pi + 6*x - sqrt(pi^2 + 12))/abs(-pi + 6*x + sqrt(pi^2 + 12)))/sqr
t(pi^2 + 12)
```

3.88 $\int \frac{1}{a+cx+bx^2} \, dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

```
[Out] (2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

Rubi [A] time = 0.0340968, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 204}

$$\frac{2\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x + b*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+cx+bx^2} \, dx &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{-4ab+c^2-x^2} \, dx, x, c+2bx\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}\end{aligned}$$

Mathematica [A] time = 0.0111292, size = 38, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x + b*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

Maple [A] time = 0.223, size = 35, normalized size = 0.9

$$2\frac{1}{\sqrt{4\,ab-c^2}}\arctan\left(\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+c*x+a),x)
```

```
[Out] 2*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))/(4*a*b-c^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09181, size = 259, normalized size = 6.82

$$\left[-\frac{\sqrt{-4\,ab+c^2}\log\left(\frac{2\,b^2x^2+2\,bcx-2\,ab+c^2-\sqrt{-4\,ab+c^2}(2\,bx+c)}{bx^2+cx+a}\right)}{4\,ab-c^2}, -\frac{2\,\arctan\left(-\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{\sqrt{4\,ab-c^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a),x, algorithm="fricas")
```

```
[Out] [-sqrt(-4*a*b + c^2)*log((2*b^2*x^2 + 2*b*c*x - 2*a*b + c^2 - sqrt(-4*a*b +
 c^2)*(2*b*x + c))/(b*x^2 + c*x + a))/(4*a*b - c^2), -2*arctan(-(2*b*x + c)
/sqrt(4*a*b - c^2))/sqrt(4*a*b - c^2)]
```

Sympy [B] time = 0.290812, size = 124, normalized size = 3.26

$$-\sqrt{-\frac{1}{4ab-c^2}}\log\left(x+\frac{-4ab\sqrt{-\frac{1}{4ab-c^2}}+c^2\sqrt{-\frac{1}{4ab-c^2}}+c}{2b}\right)+\sqrt{-\frac{1}{4ab-c^2}}\log\left(x+\frac{4ab\sqrt{-\frac{1}{4ab-c^2}}-c^2\sqrt{-\frac{1}{4ab-c^2}}+c}{2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+c*x+a),x)
```

```
[Out] -sqrt(-1/(4*a*b - c**2))*log(x + (-4*a*b*sqrt(-1/(4*a*b - c**2)) + c**2*sqr
t(-1/(4*a*b - c**2)) + c)/(2*b)) + sqrt(-1/(4*a*b - c**2))*log(x + (4*a*b*s
```

```
qrt(-1/(4*a*b - c**2)) - c**2*sqrt(-1/(4*a*b - c**2)) + c)/(2*b))
```

Giac [A] time = 1.18351, size = 46, normalized size = 1.21

$$\frac{2\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a),x, algorithm="giac")
```

```
[Out] 2*arctan((2*b*x + c)/sqrt(4*a*b - c^2))/sqrt(4*a*b - c^2)
```

3.89 $\int \frac{1}{b+2ax+bx^2}\,dx$

Optimal. Leaf size=35

$$-\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] -(ArcTanh[(a + b*x)/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])
```

Rubi [A] time = 0.0273843, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {618, 206}

$$-\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x + b*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(a + b*x)/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b+2ax+bx^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{4\left(a^2-b^2\right)-x^2}\,dx, x, 2a+2bx\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A] time = 0.0093187, size = 34, normalized size = 0.97

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*a*x + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(a + b*x)/Sqrt[-a^2 + b^2]]/Sqrt[-a^2 + b^2]
```

Maple [A] time = 0.142, size = 35, normalized size = 1.

$$\arctan\left(\frac{2\,bx+2\,a}{2}\frac{1}{\sqrt{-a^2+b^2}}\right)\frac{1}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2*a*x+b),x)
```

```
[Out] 1/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.17575, size = 261, normalized size = 7.46

$$\left[\frac{\log\left(\frac{b^2x^2+2\,abx+2\,a^2-b^2-2\,\sqrt{a^2-b^2}(bx+a)}{bx^2+2\,ax+b}\right)}{2\,\sqrt{a^2-b^2}},-\frac{\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(bx+a)}{a^2-b^2}\right)}{a^2-b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*x^2 + 2*a*b*x + 2*a^2 - b^2 - 2*sqrt(a^2 - b^2)*(b*x + a))/(b
*x^2 + 2*a*x + b))/sqrt(a^2 - b^2), -sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*x + a)/(a^2 - b^2))/(a^2 - b^2)]
```

Sympy [B] time = 0.234572, size = 100, normalized size = 2.86

$$\frac{\sqrt{\frac{1}{(a-b)(a+b)}}\log\left(x+\frac{-a^2\sqrt{\frac{1}{(a-b)(a+b)}}+a+b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2}-\frac{\sqrt{\frac{1}{(a-b)(a+b)}}\log\left(x+\frac{a^2\sqrt{\frac{1}{(a-b)(a+b)}}+a-b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2*a*x+b),x)
```

```
[Out] sqrt(1/((a - b)*(a + b)))*log(x + (-a**2*sqrt(1/((a - b)*(a + b))) + a + b*
*2*sqrt(1/((a - b)*(a + b))))/b)/2 - sqrt(1/((a - b)*(a + b)))*log(x + (a**
2*sqrt(1/((a - b)*(a + b))) + a - b**2*sqrt(1/((a - b)*(a + b))))/b)/2
```

Giac [A] time = 1.31692, size = 41, normalized size = 1.17

$$\frac{\arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b),x, algorithm="giac")
```

```
[Out] arctan((b*x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

3.90 $\int \frac{1}{b+2ax-bx^2} \, dx$

Optimal. Leaf size=32

$$-\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] -(ArcTanh[(a - b*x)/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rubi [A] time = 0.0254946, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {618, 206}

$$-\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x - b*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(a - b*x)/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{b+2ax-bx^2} \, dx &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2} \, dx, x, 2a-2bx\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} \end{aligned}$$

Mathematica [A] time = 0.0102485, size = 41, normalized size = 1.28

$$-\frac{\tan^{-1}\left(\frac{bx-a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*a*x - b*x^2)^(-1),x]
```

```
[Out] -(ArcTan[(-a + b*x)/Sqrt[-a^2 - b^2]]/Sqrt[-a^2 - b^2])
```

Maple [A] time = 0.136, size = 31, normalized size = 1.

$$\operatorname{Artanh}\left(\frac{2bx-2a}{2}\frac{1}{\sqrt{a^2+b^2}}\right)\frac{1}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+2*a*x+b),x)
```

```
[Out] 1/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*x-2*a)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.09927, size = 149, normalized size = 4.66

$$\frac{\log\left(\frac{b^2x^2-2\,abx+2\,a^2+b^2+2\,\sqrt{a^2+b^2}(bx-a)}{bx^2-2\,ax-b}\right)}{2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b),x, algorithm="fricas")
```

```
[Out] 1/2*log((b^2*x^2 - 2*a*b*x + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*(b*x - a))/(b*
x^2 - 2*a*x - b))/sqrt(a^2 + b^2)
```

Sympy [B] time = 0.323216, size = 102, normalized size = 3.19

$$-\frac{\sqrt{\frac{1}{a^2+b^2}}\log\left(x+\frac{-a^2\sqrt{\frac{1}{a^2+b^2}}-a-b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2}+\frac{\sqrt{\frac{1}{a^2+b^2}}\log\left(x+\frac{a^2\sqrt{\frac{1}{a^2+b^2}}-a+b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+2*a*x+b),x)
```

```
[Out] -sqrt(1/(a**2 + b**2))*log(x + (-a**2*sqrt(1/(a**2 + b**2)) - a - b**2*sqrt
(1/(a**2 + b**2)))/b)/2 + sqrt(1/(a**2 + b**2))*log(x + (a**2*sqrt(1/(a**2
```

```
+ b**2)) - a + b**2*sqrt(1/(a**2 + b**2)))/b)/2
```

Giac [A] time = 1.37932, size = 74, normalized size = 2.31

$$-\frac{\log\left(\frac{\left|2\,bx-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,bx-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a + 2*sqrt(a^2
+ b^2)))/sqrt(a^2 + b^2)
```

3.91 $\int \frac{1}{\left(2+4x+3x^2\right)^2} dx$

Optimal. Leaf size=43

$$\frac{3x+2}{4\left(3x^2+4x+2\right)}+\frac{3\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] (2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(4*Sqrt[2])
```

Rubi [A] time = 0.0146884, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {614, 618, 204}

$$\frac{3x+2}{4\left(3x^2+4x+2\right)}+\frac{3\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x + 3*x^2)^(-2),x]
```

```
[Out] (2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(4*Sqrt[2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(2+4x+3x^2\right)^2} dx &= \frac{2+3x}{4\left(2+4x+3x^2\right)}+\frac{3}{4}\int \frac{1}{2+4x+3x^2} dx\\
&= \frac{2+3x}{4\left(2+4x+3x^2\right)}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{-8-x^2} dx,x,4+6x\right)\\
&= \frac{2+3x}{4\left(2+4x+3x^2\right)}+\frac{3\tan^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0227275, size = 43, normalized size = 1.

$$\frac{3x+2}{4\left(3x^2+4x+2\right)}+\frac{3\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x + 3*x^2)^(-2),x]
```

```
[Out] (2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(4*Sqrt[2])
```

Maple [A] time = 0.048, size = 37, normalized size = 0.9

$$\frac{4+6\,x}{24\,x^2+32\,x+16}+\frac{3\,\sqrt{2}}{8}\arctan\left(\frac{(4+6\,x)\,\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+4*x+2)^2,x)
```

```
[Out] 1/8*(4+6*x)/(3*x^2+4*x+2)+3/8*2^(1/2)*arctan(1/4*(4+6*x)*2^(1/2))
```

Maxima [A] time = 1.70954, size = 49, normalized size = 1.14

$$\frac{3}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x+2)\right)+\frac{3\,x+2}{4\left(3\,x^2+4\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^2,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2 + 4*x + 2)
```

Fricas [A] time = 2.04242, size = 126, normalized size = 2.93

$$\frac{3\,\sqrt{2}\left(3\,x^2+4\,x+2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x+2)\right)+6\,x+4}{8\left(3\,x^2+4\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*(3*x^2 + 4*x + 2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 6*x + 4)/(
3*x^2 + 4*x + 2)
```

Sympy [A] time = 0.19527, size = 39, normalized size = 0.91

$$\frac{3x+2}{12x^2+16x+8}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{3\sqrt{2}x}{2}+\sqrt{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+4*x+2)**2,x)
```

```
[Out] (3*x + 2)/(12*x**2 + 16*x + 8) + 3*sqrt(2)*atan(3*sqrt(2)*x/2 + sqrt(2))/8
```

Giac [A] time = 1.27949, size = 49, normalized size = 1.14

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right)+\frac{3x+2}{4\left(3x^2+4x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2 + 4*x + 2)
```

3.92 $\int \frac{1}{(2+4x-3x^2)^2}\,dx$

Optimal. Leaf size=43

$$-\frac{2-3x}{20\left(-3x^2+4x+2\right)}-\frac{3\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

```
[Out] -(2 - 3*x)/(20*(2 + 4*x - 3*x^2)) - (3*ArcTanh[(2 - 3*x)/Sqrt[10]])/(20*Sqr
t[10])
```

Rubi [A] time = 0.0158066, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {614, 618, 206}

$$-\frac{2-3x}{20\left(-3x^2+4x+2\right)}-\frac{3\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x - 3*x^2)^(-2),x]
```

```
[Out] -(2 - 3*x)/(20*(2 + 4*x - 3*x^2)) - (3*ArcTanh[(2 - 3*x)/Sqrt[10]])/(20*Sqr
t[10])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx = -\frac{2-3x}{20\,(2+4x-3x^2)} + \frac{3}{20}\int \frac{1}{2+4x-3x^2}\,dx$$
$$= -\frac{2-3x}{20\,(2+4x-3x^2)} - \frac{3}{10}\,\text{Subst}\left(\int \frac{1}{40-x^2}\,dx, x, 4-6x\right)$$
$$= -\frac{2-3x}{20\,(2+4x-3x^2)} - \frac{3\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Mathematica [A] time = 0.0287392, size = 62, normalized size = 1.44

$$\frac{2-3x}{20\,(3x^2-4x-2)} - \frac{3\log\left(-3x+\sqrt{10}+2\right)}{40\sqrt{10}} + \frac{3\log\left(3x+\sqrt{10}-2\right)}{40\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x - 3*x^2)^(-2),x]
```

```
[Out] (2 - 3*x)/(20*(-2 - 4*x + 3*x^2)) - (3*Log[2 + Sqrt[10] - 3*x])/(40*Sqrt[10
]) + (3*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])
```

Maple [A] time = 0.049, size = 37, normalized size = 0.9

$$-\frac{6\,x-4}{120\,x^2-160\,x-80} + \frac{3\sqrt{10}}{200}\text{Artanh}\left(\frac{(6\,x-4)\sqrt{10}}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x+2)^2,x)
```

```
[Out] -1/40*(6*x-4)/(3*x^2-4*x-2)+3/200*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2))
```

Maxima [A] time = 2.8472, size = 63, normalized size = 1.47

$$-\frac{3}{400}\sqrt{10}\log\left(\frac{3\,x-\sqrt{10}-2}{3\,x+\sqrt{10}-2}\right) - \frac{3\,x-2}{20\,(3\,x^2-4\,x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="maxima")
```

```
[Out] -3/400*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2)) - 1/20*(3*x
 - 2)/(3*x^2 - 4*x - 2)
```

Fricas [A] time = 2.11079, size = 181, normalized size = 4.21

$$\frac{3\sqrt{10}\left(3\,x^2-4\,x-2\right)\log\left(\frac{9\,x^2+2\sqrt{10}(3\,x-2)-12\,x+14}{3\,x^2-4\,x-2}\right) - 60\,x+40}{400\,(3\,x^2-4\,x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/400*(3*sqrt(10)*(3*x^2 - 4*x - 2)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*
x + 14)/(3*x^2 - 4*x - 2)) - 60*x + 40)/(3*x^2 - 4*x - 2)
```

Sympy [A] time = 0.162571, size = 58, normalized size = 1.35

$$-\frac{3x-2}{60x^2-80x-40}+\frac{3\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400}-\frac{3\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x+2)**2,x)
```

```
[Out] -(3*x - 2)/(60*x**2 - 80*x - 40) + 3*sqrt(10)*log(x - 2/3 + sqrt(10)/3)/400
 - 3*sqrt(10)*log(x - sqrt(10)/3 - 2/3)/400
```

Giac [A] time = 1.29719, size = 69, normalized size = 1.6

$$-\frac{3}{400}\sqrt{10}\log\left(\frac{\left|6x-2\sqrt{10}-4\right|}{\left|6x+2\sqrt{10}-4\right|}\right)-\frac{3x-2}{20\left(3x^2-4x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="giac")
```

```
[Out] -3/400*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4)) -
1/20*(3*x - 2)/(3*x^2 - 4*x - 2)
```

3.93 $\int \frac{1}{\left(2+5x+3x^2\right)^2}\,dx$

Optimal. Leaf size=34

$$-\frac{6x+5}{3x^2+5x+2}+6\log(x+1)-6\log(3x+2)$$

[Out] -((5 + 6*x)/(2 + 5*x + 3*x^2)) + 6*Log[1 + x] - 6*Log[2 + 3*x]

Rubi [A] time = 0.0080393, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {614, 616, 31}

$$-\frac{6x+5}{3x^2+5x+2}+6\log(x+1)-6\log(3x+2)$$

Antiderivative was successfully verified.

[In] Int[(2 + 5*x + 3*x^2)^(-2),x]

[Out] -((5 + 6*x)/(2 + 5*x + 3*x^2)) + 6*Log[1 + x] - 6*Log[2 + 3*x]

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(2+5x+3x^2\right)^2}\,dx &= -\frac{5+6x}{2+5x+3x^2}-6\int \frac{1}{2+5x+3x^2}\,dx\\ &= -\frac{5+6x}{2+5x+3x^2}-18\int \frac{1}{2+3x}\,dx+18\int \frac{1}{3+3x}\,dx\\ &= -\frac{5+6x}{2+5x+3x^2}+6\log(1+x)-6\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0113213, size = 33, normalized size = 0.97

$$\frac{-6x-5}{3x^2+5x+2}+6\log(x+1)-6\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + 3*x^2)^(-2),x]
```

```
[Out] (-5 - 6*x)/(2 + 5*x + 3*x^2) + 6*Log[1 + x] - 6*Log[2 + 3*x]
```

Maple [A] time = 0.056, size = 32, normalized size = 0.9

$$-(1+x)^{-1}+6\ln(1+x)-3(2+3x)^{-1}-6\ln(2+3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+5*x+2)^2,x)
```

```
[Out] -1/(1+x)+6*ln(1+x)-3/(2+3*x)-6*ln(2+3*x)
```

Maxima [A] time = 1.90578, size = 46, normalized size = 1.35

$$-\frac{6x+5}{3x^2+5x+2}-6\log(3x+2)+6\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^2,x, algorithm="maxima")
```

```
[Out] -(6*x + 5)/(3*x^2 + 5*x + 2) - 6*log(3*x + 2) + 6*log(x + 1)
```

Fricas [A] time = 2.15157, size = 132, normalized size = 3.88

$$-\frac{6\left(3x^2+5x+2\right)\log(3x+2)-6\left(3x^2+5x+2\right)\log(x+1)+6x+5}{3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^2,x, algorithm="fricas")
```

```
[Out] -(6*(3*x^2 + 5*x + 2)*log(3*x + 2) - 6*(3*x^2 + 5*x + 2)*log(x + 1) + 6*x +
 5)/(3*x^2 + 5*x + 2)
```

Sympy [A] time = 0.266646, size = 29, normalized size = 0.85

$$-\frac{6x+5}{3x^2+5x+2}-6\log\left(x+\frac{2}{3}\right)+6\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+5*x+2)**2,x)
```

```
[Out] -(6*x + 5)/(3*x**2 + 5*x + 2) - 6*log(x + 2/3) + 6*log(x + 1)
```

Giac [A] time = 1.27257, size = 49, normalized size = 1.44

$$-\frac{6x+5}{3x^2+5x+2}-6\log(|3x+2|)+6\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^2,x, algorithm="giac")
```

```
[Out] -(6*x + 5)/(3*x^2 + 5*x + 2) - 6*log(abs(3*x + 2)) + 6*log(abs(x + 1))
```

3.94 $\int \frac{1}{\left(2+5x-3x^2\right)^2} \, dx$

Optimal. Leaf size=42

$$-\frac{5-6x}{49\left(-3x^2+5x+2\right)}-\frac{6}{343}\log(2-x)+\frac{6}{343}\log(3x+1)$$

```
[Out] -(5 - 6*x)/(49*(2 + 5*x - 3*x^2)) - (6*Log[2 - x])/343 + (6*Log[1 + 3*x])/3
43
```

Rubi [A] time = 0.0090168, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {614, 616, 31}

$$-\frac{5-6x}{49\left(-3x^2+5x+2\right)}-\frac{6}{343}\log(2-x)+\frac{6}{343}\log(3x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x - 3*x^2)^(-2),x]
```

```
[Out] -(5 - 6*x)/(49*(2 + 5*x - 3*x^2)) - (6*Log[2 - x])/343 + (6*Log[1 + 3*x])/3
43
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(2+5x-3x^2\right)^2} \, dx &= -\frac{5-6x}{49\left(2+5x-3x^2\right)}+\frac{6}{49}\int \frac{1}{2+5x-3x^2} \, dx\\
&= -\frac{5-6x}{49\left(2+5x-3x^2\right)}-\frac{18}{343}\int \frac{1}{-1-3x} \, dx+\frac{18}{343}\int \frac{1}{6-3x} \, dx\\
&= -\frac{5-6x}{49\left(2+5x-3x^2\right)}-\frac{6}{343}\log(2-x)+\frac{6}{343}\log(1+3x)
\end{aligned}$$

Mathematica [A] time = 0.0127719, size = 42, normalized size = 1.

$$\frac{5-6x}{49\left(3x^2-5x-2\right)}-\frac{6}{343}\log(2-x)+\frac{6}{343}\log(3x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x - 3*x^2)^(-2),x]
```

```
[Out] (5 - 6*x)/(49*(-2 - 5*x + 3*x^2)) - (6*Log[2 - x])/343 + (6*Log[1 + 3*x])/3
43
```

Maple [A] time = 0.056, size = 32, normalized size = 0.8

$$-\frac{3}{49+147\,x}+\frac{6\,\ln\left(1+3\,x\right)}{343}-\frac{1}{-98+49\,x}-\frac{6\,\ln\left(-2+x\right)}{343}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+5*x+2)^2,x)
```

```
[Out] -3/49/(1+3*x)+6/343*ln(1+3*x)-1/49/(-2+x)-6/343*ln(-2+x)
```

Maxima [A] time = 1.18605, size = 46, normalized size = 1.1

$$-\frac{6\,x-5}{49\left(3\,x^2-5\,x-2\right)}+\frac{6}{343}\,\log\left(3\,x+1\right)-\frac{6}{343}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="maxima")
```

```
[Out] -1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*log(3*x + 1) - 6/343*log(x - 2)
```

Fricas [A] time = 2.05154, size = 142, normalized size = 3.38

$$\frac{6\left(3\,x^2-5\,x-2\right)\log\left(3\,x+1\right)-6\left(3\,x^2-5\,x-2\right)\log\left(x-2\right)-42\,x+35}{343\left(3\,x^2-5\,x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/343*(6*(3*x^2 - 5*x - 2)*log(3*x + 1) - 6*(3*x^2 - 5*x - 2)*log(x - 2) -
42*x + 35)/(3*x^2 - 5*x - 2)
```

Sympy [A] time = 0.191956, size = 32, normalized size = 0.76

$$-\frac{6x-5}{147x^2-245x-98}-\frac{6\log\left(x-2\right)}{343}+\frac{6\log\left(x+\frac{1}{3}\right)}{343}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+5*x+2)**2,x)
```

```
[Out] -(6*x - 5)/(147*x**2 - 245*x - 98) - 6*log(x - 2)/343 + 6*log(x + 1/3)/343
```

Giac [A] time = 1.23187, size = 49, normalized size = 1.17

$$-\frac{6\,x-5}{49\,\left(3\,x^2-5\,x-2\right)}+\frac{6}{343}\,\log\left(|3\,x+1|\right)-\frac{6}{343}\,\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="giac")
```

```
[Out] -1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*log(abs(3*x + 1)) - 6/343*log(abs
(x - 2))
```

3.95 $\int \frac{1}{\left(a+cx+bx^2\right)^2} dx$

Optimal. Leaf size=71

$$\frac{2bx+c}{\left(4ab-c^2\right)\left(a+bx^2+cx\right)}+\frac{4b\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\left(4ab-c^2\right)^{3/2}}$$

```
[Out] (c + 2*b*x)/((4*a*b - c^2)*(a + c*x + b*x^2)) + (4*b*ArcTan[(c + 2*b*x)/Sqr
t[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

Rubi [A] time = 0.0397878, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {614, 618, 204}

$$\frac{2bx+c}{\left(4ab-c^2\right)\left(a+bx^2+cx\right)}+\frac{4b\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\left(4ab-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x + b*x^2)^(-2),x]
```

```
[Out] (c + 2*b*x)/((4*a*b - c^2)*(a + c*x + b*x^2)) + (4*b*ArcTan[(c + 2*b*x)/Sqr
t[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+cx+bx^2\right)^2}\,dx = \frac{c+2bx}{\left(4ab-c^2\right)\left(a+cx+bx^2\right)} + \frac{(2b)\int \frac{1}{a+cx+bx^2}\,dx}{4ab-c^2}$$

$$= \frac{c+2bx}{\left(4ab-c^2\right)\left(a+cx+bx^2\right)} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{-4ab+c^2-x^2}\,dx, x, c+2bx\right)}{4ab-c^2}$$

$$= \frac{c+2bx}{\left(4ab-c^2\right)\left(a+cx+bx^2\right)} + \frac{4b\tan^{-1}\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{\left(4ab-c^2\right)^{3/2}}$$

Mathematica [A] time = 0.0651828, size = 70, normalized size = 0.99

$$\frac{2bx+c}{\left(4ab-c^2\right)(a+x(bx+c))} + \frac{4b\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\left(4ab-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x + b*x^2)^(-2),x]
```

```
[Out] (c + 2*b*x)/((4*a*b - c^2)*(a + x*(c + b*x))) + (4*b*ArcTan[(c + 2*b*x)/Sqr
t[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

Maple [A] time = 0.191, size = 68, normalized size = 1.

$$\frac{2\,bx+c}{\left(4\,ab-c^2\right)\left(bx^2+cx+a\right)} + 4\,\frac{b}{\left(4\,ab-c^2\right)^{3/2}}\arctan\left(\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+c*x+a)^2,x)
```

```
[Out] (2*b*x+c)/(4*a*b-c^2)/(b*x^2+c*x+a)+4*b*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))
/(4*a*b-c^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19871, size = 724, normalized size = 10.2

$$\left[\frac{4\,abc-c^3+2\left(b^2x^2+bcx+ab\right)\sqrt{-4\,ab+c^2}\log\left(\frac{2\,b^2x^2+2\,bcx-2\,ab+c^2+\sqrt{-4\,ab+c^2}(2\,bx+c)}{bx^2+cx+a}\right)+2\left(4\,ab^2-bc^2\right)x}{16\,a^3b^2-8\,a^2bc^2+ac^4+\left(16\,a^2b^3-8\,ab^2c^2+bc^4\right)x^2+\left(16\,a^2b^2c-8\,abc^3+c^5\right)x}, \frac{4\,abc-c^3-4}{16\,a^3b^2-8\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a)^2,x, algorithm="fricas")
```

```
[Out] [(4*a*b*c - c^3 + 2*(b^2*x^2 + b*c*x + a*b)*sqrt(-4*a*b + c^2)*log((2*b^2*x
^2 + 2*b*c*x - 2*a*b + c^2 + sqrt(-4*a*b + c^2)*(2*b*x + c))/(b*x^2 + c*x +
 a)) + 2*(4*a*b^2 - b*c^2)*x)/(16*a^3*b^2 - 8*a^2*b*c^2 + a*c^4 + (16*a^2*b
^3 - 8*a*b^2*c^2 + b*c^4)*x^2 + (16*a^2*b^2*c - 8*a*b*c^3 + c^5)*x), (4*a*b
*c - c^3 - 4*(b^2*x^2 + b*c*x + a*b)*sqrt(4*a*b - c^2)*arctan(-(2*b*x + c)/
sqrt(4*a*b - c^2)) + 2*(4*a*b^2 - b*c^2)*x)/(16*a^3*b^2 - 8*a^2*b*c^2 + a*c
^4 + (16*a^2*b^3 - 8*a*b^2*c^2 + b*c^4)*x^2 + (16*a^2*b^2*c - 8*a*b*c^3 + c
^5)*x)]
```

Sympy [B] time = 1.17108, size = 265, normalized size = 3.73

$$-2b\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}\log\left(x+\frac{-32a^2b^3\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}+16ab^2c^2\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}-2bc^4\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}+2bc}{4b^2}\right)+2b\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+c*x+a)**2,x)
```

```
[Out] -2*b*sqrt(-1/(4*a*b - c**2)**3)*log(x + (-32*a**2*b**3*sqrt(-1/(4*a*b - c**
2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*b - c**2)**3) - 2*b*c**4*sqrt(-1/(4*a*
b - c**2)**3) + 2*b*c)/(4*b**2)) + 2*b*sqrt(-1/(4*a*b - c**2)**3)*log(x + (
32*a**2*b**3*sqrt(-1/(4*a*b - c**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*b - c
**2)**3) + 2*b*c**4*sqrt(-1/(4*a*b - c**2)**3) + 2*b*c)/(4*b**2)) + (2*b*x
+ c)/(4*a**2*b - a*c**2 + x**2*(4*a*b**2 - b*c**2) + x*(4*a*b*c - c**3))
```

Giac [A] time = 1.24021, size = 90, normalized size = 1.27

$$\frac{4\,b\arctan\left(\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{\left(4\,ab-c^2\right)^{\frac{3}{2}}}+\frac{2\,bx+c}{\left(bx^2+cx+a\right)\left(4\,ab-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a)^2,x, algorithm="giac")
```

```
[Out] 4*b*arctan((2*b*x + c)/sqrt(4*a*b - c^2))/(4*a*b - c^2)^(3/2) + (2*b*x + c)
/((b*x^2 + c*x + a)*(4*a*b - c^2))
```

3.96 $\int \frac{1}{\left(b+2ax+bx^2\right)^2}\,dx$

Optimal. Leaf size=72

$$\frac{b\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}-\frac{a+bx}{2\left(a^2-b^2\right)\left(2ax+bx^2+b\right)}$$

```
[Out] -(a + b*x)/(2*(a^2 - b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTanh[(a + b*x)/Sqrt[
a^2 - b^2]])/(2*(a^2 - b^2)^(3/2))
```

Rubi [A] time = 0.0355411, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {614, 618, 206}

$$\frac{b\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}-\frac{a+bx}{2\left(a^2-b^2\right)\left(2ax+bx^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x + b*x^2)^(-2),x]
```

```
[Out] -(a + b*x)/(2*(a^2 - b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTanh[(a + b*x)/Sqrt[
a^2 - b^2]])/(2*(a^2 - b^2)^(3/2))
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(b+2ax+bx^2\right)^2}\,dx = -\frac{a+bx}{2\left(a^2-b^2\right)\left(b+2ax+bx^2\right)} - \frac{b\int \frac{1}{b+2ax+bx^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{a+bx}{2\left(a^2-b^2\right)\left(b+2ax+bx^2\right)} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{4\left(a^2-b^2\right)-x^2}\,dx, x, 2a+2bx\right)}{a^2-b^2}$$

$$= -\frac{a+bx}{2\left(a^2-b^2\right)\left(b+2ax+bx^2\right)} + \frac{b\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}$$

Mathematica [A] time = 0.0427679, size = 72, normalized size = 1.

$$\frac{a+bx}{2\left(b^2-a^2\right)\left(2ax+bx^2+b\right)} + \frac{b\tan^{-1}\left(\frac{a+bx}{\sqrt{b^2-a^2}}\right)}{2\left(b^2-a^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*a*x + b*x^2)^(-2),x]
```

```
[Out] (a + b*x)/(2*(-a^2 + b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTan[(a + b*x)/Sqrt[-
a^2 + b^2]])/(2*(-a^2 + b^2)^(3/2))
```

Maple [A] time = 0.152, size = 86, normalized size = 1.2

$$\frac{2\,bx+2\,a}{\left(-4\,a^2+4\,b^2\right)\left(bx^2+2\,ax+b\right)} + 2\,\frac{b}{\left(-4\,a^2+4\,b^2\right)\sqrt{-a^2+b^2}}\arctan\left(1/2\,\frac{2\,bx+2\,a}{\sqrt{-a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2*a*x+b)^2,x)
```

```
[Out] (2*b*x+2*a)/(-4*a^2+4*b^2)/(b*x^2+2*a*x+b)+2*b/(-4*a^2+4*b^2)/(-a^2+b^2)^(1
/2)*arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.41407, size = 664, normalized size = 9.22

$$\left[-\frac{2\,a^3-2\,ab^2+\left(b^2x^2+2\,abx+b^2\right)\sqrt{a^2-b^2}\log\left(\frac{b^2x^2+2\,abx+2\,a^2-b^2-2\,\sqrt{a^2-b^2}(bx+a)}{bx^2+2\,ax+b}\right)+2\left(a^2b-b^3\right)x}{4\left(a^4b-2\,a^2b^3+b^5+\left(a^4b-2\,a^2b^3+b^5\right)x^2+2\left(a^5-2\,a^3b^2+ab^4\right)x\right)},-\frac{a^3-ab^2-\left(b^2x^2+\right.}{2\left(a^4b-2\,a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a^3 - 2*a*b^2 + (b^2*x^2 + 2*a*b*x + b^2)*sqrt(a^2 - b^2)*log((b^2
*x^2 + 2*a*b*x + 2*a^2 - b^2 - 2*sqrt(a^2 - b^2)*(b*x + a))/(b*x^2 + 2*a*x
+ b)) + 2*(a^2*b - b^3)*x)/(a^4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 +
b^5)*x^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x), -1/2*(a^3 - a*b^2 - (b^2*x^2 + 2
*a*b*x + b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*x + a)/(a^2 - b^
2)) + (a^2*b - b^3)*x)/(a^4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 + b^5)
*x^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x)]
```

Sympy [B] time = 1.39258, size = 228, normalized size = 3.17

$$-\frac{b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}\log\left(x+\frac{-a^4b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+2a^2b^3\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+ab-b^5\sqrt{\frac{1}{(a-b)^3(a+b)^3}}}{b^2}\right)}{4}+\frac{b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}\log\left(x+\frac{a^4b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}-2a^2b\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2*a*x+b)**2,x)
```

```
[Out] -b*sqrt(1/((a - b)**3*(a + b)**3))*log(x + (-a**4*b*sqrt(1/((a - b)**3*(a +
 b)**3)) + 2*a**2*b**3*sqrt(1/((a - b)**3*(a + b)**3)) + a*b - b**5*sqrt(1/
((a - b)**3*(a + b)**3)))/b**2)/4 + b*sqrt(1/((a - b)**3*(a + b)**3))*log(x
 + (a**4*b*sqrt(1/((a - b)**3*(a + b)**3)) - 2*a**2*b**3*sqrt(1/((a - b)**3
*(a + b)**3)) + a*b + b**5*sqrt(1/((a - b)**3*(a + b)**3)))/b**2)/4 - (a +
b*x)/(2*a**2*b - 2*b**3 + x**2*(2*a**2*b - 2*b**3) + x*(4*a**3 - 4*a*b**2))
```

Giac [A] time = 1.19746, size = 101, normalized size = 1.4

$$-\frac{b\arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{2\left(a^2-b^2\right)\sqrt{-a^2+b^2}}-\frac{bx+a}{2\left(bx^2+2\,ax+b\right)\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((b*x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2)) -
1/2*(b*x + a)/((b*x^2 + 2*a*x + b)*(a^2 - b^2))
```

3.97 $\int \frac{1}{\left(b+2ax-bx^2\right)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{a-bx}{2\left(a^2+b^2\right)\left(2ax-bx^2+b\right)}-\frac{b\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -(a - b*x)/(2*(a^2 + b^2)*(b + 2*a*x - b*x^2)) - (b*ArcTanh[(a - b*x)/Sqrt[
a^2 + b^2]])/(2*(a^2 + b^2)^(3/2))
```

Rubi [A] time = 0.0336722, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {614, 618, 206}

$$-\frac{a-bx}{2\left(a^2+b^2\right)\left(2ax-bx^2+b\right)}-\frac{b\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x - b*x^2)^(-2),x]
```

```
[Out] -(a - b*x)/(2*(a^2 + b^2)*(b + 2*a*x - b*x^2)) - (b*ArcTanh[(a - b*x)/Sqrt[
a^2 + b^2]])/(2*(a^2 + b^2)^(3/2))
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(b+2ax-bx^2\right)^2}\,dx = -\frac{a-bx}{2\left(a^2+b^2\right)\left(b+2ax-bx^2\right)} + \frac{b\int \frac{1}{b+2ax-bx^2}\,dx}{2\left(a^2+b^2\right)}$$

$$= -\frac{a-bx}{2\left(a^2+b^2\right)\left(b+2ax-bx^2\right)} - \frac{b\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2a-2bx\right)}{a^2+b^2}$$

$$= -\frac{a-bx}{2\left(a^2+b^2\right)\left(b+2ax-bx^2\right)} - \frac{b\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}$$

Mathematica [A] time = 0.0612156, size = 78, normalized size = 1.13

$$\frac{\frac{bx-a}{2ax-bx^2+b} - \frac{b\tan^{-1}\left(\frac{bx-a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*a*x - b*x^2)^(-2),x]
```

```
[Out] ((-a + b*x)/(b + 2*a*x - b*x^2) - (b*ArcTan[(-a + b*x)/Sqrt[-a^2 - b^2]])/S
qrt[-a^2 - b^2])/(2*(a^2 + b^2))
```

Maple [A] time = 0.149, size = 84, normalized size = 1.2

$$\frac{2\,bx-2\,a}{\left(-4\,a^2-4\,b^2\right)\left(bx^2-2\,ax-b\right)} - 2\,\frac{b}{\left(-4\,a^2-4\,b^2\right)\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,bx-2\,a}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+2*a*x+b)^2,x)
```

```
[Out] (2*b*x-2*a)/(-4*a^2-4*b^2)/(b*x^2-2*a*x-b)-2*b/(-4*a^2-4*b^2)/(a^2+b^2)^(1/
2)*arctanh(1/2*(2*b*x-2*a)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.50362, size = 360, normalized size = 5.22

$$-\frac{2\,a^3+2\,ab^2+\left(b^2x^2-2\,abx-b^2\right)\sqrt{a^2+b^2}\log\left(\frac{b^2x^2-2\,abx+2\,a^2+b^2+2\,\sqrt{a^2+b^2}(bx-a)}{bx^2-2\,ax-b}\right)-2\left(a^2b+b^3\right)x}{4\left(a^4b+2\,a^2b^3+b^5-\left(a^4b+2\,a^2b^3+b^5\right)x^2+2\left(a^5+2\,a^3b^2+ab^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^3 + 2*a*b^2 + (b^2*x^2 - 2*a*b*x - b^2)*sqrt(a^2 + b^2)*log((b^2*
x^2 - 2*a*b*x + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*(b*x - a))/(b*x^2 - 2*a*x -
 b)) - 2*(a^2*b + b^3)*x)/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b
^5)*x^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*x)
```

Sympy [B] time = 1.03575, size = 218, normalized size = 3.16

$$-\frac{b\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}\log\left(x+\frac{-a^4b\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}-2a^2b^3\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}-ab-b^5\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}}{b^2}\right)}{4}+\frac{b\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}\log\left(x+\frac{a^4b\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}+2a^2b^3\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}-ab+b^5\sqrt{\frac{1}{(a^2}\cdots}}{b^2}\cdots}{4}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+2*a*x+b)**2,x)
```

```
[Out] -b*sqrt((a**2 + b**2)**(-3))*log(x + (-a**4*b*sqrt((a**2 + b**2)**(-3)) - 2
*a**2*b**3*sqrt((a**2 + b**2)**(-3)) - a*b - b**5*sqrt((a**2 + b**2)**(-3))
)/b**2)/4 + b*sqrt((a**2 + b**2)**(-3))*log(x + (a**4*b*sqrt((a**2 + b**2)*
*(-3)) + 2*a**2*b**3*sqrt((a**2 + b**2)**(-3)) - a*b + b**5*sqrt((a**2 + b*
*2)**(-3)))/b**2)/4 - (-a + b*x)/(-2*a**2*b - 2*b**3 + x**2*(2*a**2*b + 2*b
**3) + x*(-4*a**3 - 4*a*b**2))
```

Giac [A] time = 1.21631, size = 122, normalized size = 1.77

$$-\frac{b\log\left(\frac{\left|2\,bx-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,bx-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{4\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{bx-a}{2\left(bx^2-2\,ax-b\right)\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="giac")
```

```
[Out] -1/4*b*log(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a + 2*sqrt(a^
2 + b^2)))/(a^2 + b^2)^(3/2) - 1/2*(b*x - a)/((b*x^2 - 2*a*x - b)*(a^2 + b^
2))
```

3.98 $\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}} x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx$

Optimal. Leaf size=62

$$-\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\tan^{-1}\left(\cot\left(\frac{\pi-2\pi k}{n}\right)-x\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\right)$$

```
[Out] -((ArcTan[Cot[(Pi - 2*k*Pi)/n] - (x*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1)]*Csc
[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1))
```

Rubi [A] time = 0.157198, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {618, 204}

$$-\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\tan^{-1}\left(\cot\left(\frac{\pi-2\pi k}{n}\right)-x\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-1),x]
```

```
[Out] -((ArcTan[Cot[(Pi - 2*k*Pi)/n] - (x*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1)]*Csc
[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}} x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx=-\left(2\,\text{Subst}\left(\int \frac{1}{-x^2-4\left(\frac{a}{b}\right)^{2/n}\left(1-\cos^2\left(\frac{\pi-2k\pi}{n}\right)\right)}\,dx,x,2x-2\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi\right.$$

$$=-\left(\frac{a}{b}\right)^{-1/n}\tan^{-1}\left(\cot\left(\frac{\pi-2k\pi}{n}\right)-\left(\frac{a}{b}\right)^{-1/n}x\csc\left(\frac{\pi-2k\pi}{n}\right)\right)\csc\left(\frac{\pi-2k\pi}{n}\right.$$

Mathematica [A] time = 0.0948207, size = 65, normalized size = 1.05

$$\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\tan^{-1}\left(\frac{\tan\left(\frac{\pi-2\pi k}{2n}\right)\left(\left(\frac{a}{b}\right)^{\frac{1}{n}}+x\right)}{\left(\frac{a}{b}\right)^{\frac{1}{n}}-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-1),
x]
```

```
[Out] (ArcTan[(((a/b)^n^(-1) + x)*Tan[(Pi - 2*k*Pi)/(2*n)])/((a/b)^n^(-1) - x)]*C
sc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1)
```

Maple [A] time = 0.423, size = 111, normalized size = 1.8

$$\arctan\left(\frac{1}{2}\left(2x-2\sqrt[n]{\frac{a}{b}}\cos\left(\frac{(2k-1)\pi}{n}\right)\right)\frac{1}{\sqrt{-\left(\sqrt[n]{\frac{a}{b}}\right)^{2}\left(\cos\left(\frac{(2k-1)\pi}{n}\right)\right)^{2}+\left(\frac{a}{b}\right)^{2n^{-1}}}}\right)\frac{1}{\sqrt{-\left(\sqrt[n]{\frac{a}{b}}\right)^{2}\left(\cos\left(\frac{(2k-1)\pi}{n}\right)\right)^{2}+\left(\frac{a}{b}\right)^{2n^{-1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/b*a)^(2/n)+x^2-2*(1/b*a)^(1/n)*x*cos((-2*Pi*k+Pi)/n)),x)
```

```
[Out] 1/(-((1/b*a)^(1/n))^2*cos(Pi*(2*k-1)/n)^2+(1/b*a)^(2/n))^(1/2)*arctan(1/2*(
2*x-2*(1/b*a)^(1/n)*cos(Pi*(2*k-1)/n))/(-((1/b*a)^(1/n))^2*cos(Pi*(2*k-1)/n
)^2+(1/b*a)^(2/n))^(1/2))
```

Maxima [B] time = 1.7059, size = 215, normalized size = 3.47

$$\frac{\log\left(\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+\sqrt{\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^{2}-1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}-x}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-\sqrt{\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^{2}-1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}-x}\right)}{2\sqrt{\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^{2}-1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algori
thm="maxima")
```

```
[Out] 1/2*log(((a/b)^(1/n)*cos(2*pi*k/n - pi/n) + sqrt(cos(2*pi*k/n - pi/n)^2 - 1
)*(a/b)^(1/n) - x)/((a/b)^(1/n)*cos(2*pi*k/n - pi/n) - sqrt(cos(2*pi*k/n -
pi/n)^2 - 1)*(a/b)^(1/n) - x))/(sqrt(cos(2*pi*k/n - pi/n)^2 - 1)*(a/b)^(1/n
))
```

Fricas [A] time = 2.67609, size = 161, normalized size = 2.6

$$-\frac{\arctan\left(\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-x}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\sin\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)}\right)}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\sin\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algori
thm="fricas")
```

```
[Out] -arctan(((a/b)^(1/n)*cos(2*pi*k/n - pi/n) - x)/((a/b)^(1/n)*sin(2*pi*k/n -
pi/n)))/((a/b)^(1/n)*sin(2*pi*k/n - pi/n))
```

Sympy [B] time = 1.40133, size = 212, normalized size = 3.42

$$-\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^{2}\left(\frac{\pi(2k-1)}{n}\right)-1}}\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^{2}\left(\frac{\pi(2k-1)}{n}\right)-1}}\left(-2\left(\frac{a}{b}\right)^{\frac{2}{n}}\cos^{2}\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+2\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)}{2}\right)}{2}+\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^{2}\left(\frac{\pi(2k-1)}{n}\right)-1}}\log\left(x-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/b)**(2/n)+x**2-2*(a/b)**(1/n)*x*cos((-2*pi*k+pi)/n)),x)
```

```
[Out] -sqrt((a/b)**(-2/n)/(cos(pi*(2*k - 1)/n)**2 - 1))*log(x - (a/b)**(1/n)*cos(
2*pi*k/n - pi/n) - sqrt((a/b)**(-2/n)/(cos(pi*(2*k - 1)/n)**2 - 1))*(-2*(a/
b)**(2/n)*cos(2*pi*k/n - pi/n)**2 + 2*(a/b)**(2/n))/2)/2 + sqrt((a/b)**(-2/
n)/(cos(pi*(2*k - 1)/n)**2 - 1))*log(x - (a/b)**(1/n)*cos(2*pi*k/n - pi/n)
+ sqrt((a/b)**(-2/n)/(cos(pi*(2*k - 1)/n)**2 - 1))*(-2*(a/b)**(2/n)*cos(2*p
i*k/n - pi/n)**2 + 2*(a/b)**(2/n))/2)/2
```

Giac [A] time = 1.3673, size = 135, normalized size = 2.18

$$\frac{\arctan\left(-\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(-\frac{2\pi k}{n}+\frac{\pi}{n}\right)-x}{\sqrt{-\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^{2}+1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}\right)}{\sqrt{-\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^{2}+1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algori
thm="giac")
```

```
[Out] arctan(-((a/b)^(1/n)*cos(-2*pi*k/n + pi/n) - x)/(sqrt(-cos(2*pi*k/n - pi/n)
^2 + 1)*(a/b)^(1/n)))/(sqrt(-cos(2*pi*k/n - pi/n)^2 + 1)*(a/b)^(1/n))
```

3.99 $\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx$

Optimal. Leaf size=33

$$\frac{2\tanh^{-1}\left(\frac{2b^2x-\sqrt{b^2-4ab^3}}{b}\right)}{b}$$

```
[Out] (2*ArcTanh[(-Sqrt[b^2 - 4*a*b^3] + 2*b^2*x)/b])/b
```

Rubi [A] time = 0.0357354, antiderivative size = 58, normalized size of antiderivative = 1.76, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {616, 31}

$$\frac{\log\left(-\sqrt{b^2-4ab^3}+2b^2x+b\right)}{b}-\frac{\log\left(\sqrt{b^2-4ab^3}-2b^2x+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

```
[Out] -(Log[b + Sqrt[b^2 - 4*a*b^3] - 2*b^2*x]/b) + Log[b - Sqrt[b^2 - 4*a*b^3] +
 2*b^2*x]/b
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx &= -\left(b\int \frac{1}{\frac{1}{2}\left(-b+\sqrt{b^2-4ab^3}\right)-b^2x}\,dx\right)+b\int \frac{1}{\frac{1}{2}\left(b+\sqrt{b^2-4ab^3}\right)-b^2x}\,dx\\ &= -\frac{\log\left(b+\sqrt{b^2-4ab^3}-2b^2x\right)}{b}+\frac{\log\left(b-\sqrt{b^2-4ab^3}+2b^2x\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0324871, size = 34, normalized size = 1.03

$$\frac{2\tanh^{-1}\left(\frac{2b^2x-\sqrt{-b^2(4ab-1)}}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

```
[Out] (2*ArcTanh[(-Sqrt[-(b^2*(-1 + 4*a*b))] + 2*b^2*x)/b])/b
```

Maple [A] time = 0.178, size = 31, normalized size = 0.9

$$-2\,\frac{1}{b}\mathrm{Artanh}\left(\frac{-2\,b^2x+\sqrt{-b^2\,(4\,ab-1)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x)
```

```
[Out] -2/b*arctanh((-2*b^2*x+(-b^2*(4*a*b-1))^(1/2))/b)
```

Maxima [B] time = 1.14659, size = 88, normalized size = 2.67

$$-\frac{\log\left(\frac{2\,b^2x-\sqrt{-4\,ab^3+b^2}-\sqrt{b^2}}{2\,b^2x-\sqrt{-4\,ab^3+b^2}+\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log((2*b^2*x - sqrt(-4*a*b^3 + b^2) - sqrt(b^2))/(2*b^2*x - sqrt(-4*a*b^3
+ b^2) + sqrt(b^2)))/sqrt(b^2)
```

Fricas [B] time = 2.70566, size = 128, normalized size = 3.88

$$\frac{\log\left(\frac{2\,b^2x+b-\sqrt{-4\,ab^3+b^2}}{b}\right)-\log\left(\frac{2\,b^2x-b-\sqrt{-4\,ab^3+b^2}}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] (log((2*b^2*x + b - sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b - sqrt(-4*a
*b^3 + b^2))/b))/b
```

Sympy [B] time = 0.570447, size = 56, normalized size = 1.7

$$-\frac{\log\left(x-\frac{1}{2b}-\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)-\log\left(x+\frac{1}{2b}-\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b**2*x**2+x*(-4*a*b**3+b**2)**(1/2)),x)
```

```
[Out] -(log(x - 1/(2*b) - sqrt(-4*a*b**3 + b**2)/(2*b**2)) - log(x + 1/(2*b) - sq
rt(-4*a*b**3 + b**2)/(2*b**2)))/b
```

Giac [A] time = 1.26446, size = 76, normalized size = 2.3

$$-\frac{\log\left(\frac{\left|2\,b^2x-\sqrt{-4\,ab+1}|b|-|b|\right|}{\left|2\,b^2x-\sqrt{-4\,ab+1}|b|+|b|\right|}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="giac")
```

```
[Out] -log(abs(2*b^2*x - sqrt(-4*a*b + 1)*abs(b) - abs(b))/abs(2*b^2*x - sqrt(-4*
a*b + 1)*abs(b) + abs(b)))/abs(b)
```

3.100 $\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx$

Optimal. Leaf size=31

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b^2-4ab^3}+2b^2x}{b}\right)}{b}$$

```
[Out] (2*ArcTanh[(Sqrt[b^2 - 4*a*b^3] + 2*b^2*x)/b])/b
```

Rubi [A] time = 0.0291565, antiderivative size = 58, normalized size of antiderivative = 1.87, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {616, 31}

$$\frac{\log\left(\sqrt{b^2-4ab^3}+2b^2x+b\right)}{b}-\frac{\log\left(-\sqrt{b^2-4ab^3}-2b^2x+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

```
[Out] -(Log[b - Sqrt[b^2 - 4*a*b^3] - 2*b^2*x]/b) + Log[b + Sqrt[b^2 - 4*a*b^3] +
 2*b^2*x]/b
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx &= -\left(b\int \frac{1}{\frac{1}{2}\left(-b-\sqrt{b^2-4ab^3}\right)-b^2x}\,dx\right)+b\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ab^3}\right)-b^2x}\,dx\\ &= -\frac{\log\left(b-\sqrt{b^2-4ab^3}-2b^2x\right)}{b}+\frac{\log\left(b+\sqrt{b^2-4ab^3}+2b^2x\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0311477, size = 32, normalized size = 1.03

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-b^2(4ab-1)}+2b^2x}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-(b^2*(-1 + 4*a*b))] + 2*b^2*x)/b])/b
```

Maple [A] time = 0.147, size = 31, normalized size = 1.

$$2\frac{1}{b}\text{Artanh}\left(\frac{2b^2x+\sqrt{-b^2(4ab-1)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x)
```

```
[Out] 2/b*arctanh((2*b^2*x+(-b^2*(4*a*b-1))^(1/2))/b)
```

Maxima [B] time = 1.22274, size = 82, normalized size = 2.65

$$-\frac{\log\left(\frac{2b^2x+\sqrt{-4ab^3+b^2}-\sqrt{b^2}}{2b^2x+\sqrt{-4ab^3+b^2}+\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log((2*b^2*x + sqrt(-4*a*b^3 + b^2) - sqrt(b^2))/(2*b^2*x + sqrt(-4*a*b^3
+ b^2) + sqrt(b^2)))/sqrt(b^2)
```

Fricas [B] time = 2.3588, size = 128, normalized size = 4.13

$$\frac{\log\left(\frac{2b^2x+b+\sqrt{-4ab^3+b^2}}{b}\right)-\log\left(\frac{2b^2x-b+\sqrt{-4ab^3+b^2}}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] (log((2*b^2*x + b + sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b + sqrt(-4*a
*b^3 + b^2))/b))/b
```

Sympy [B] time = 0.450968, size = 56, normalized size = 1.81

$$-\frac{\log\left(x-\frac{1}{2b}+\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)-\log\left(x+\frac{1}{2b}+\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b**2*x**2-x*(-4*a*b**3+b**2)**(1/2)),x)
```

```
[Out] -(log(x - 1/(2*b) + sqrt(-4*a*b**3 + b**2)/(2*b**2)) - log(x + 1/(2*b) + sq
rt(-4*a*b**3 + b**2)/(2*b**2)))/b
```

Giac [A] time = 1.21375, size = 73, normalized size = 2.35

$$-\frac{\log\left(\frac{\left|2\,b^2x+\sqrt{-4\,ab+1}|b|-|b|\right|}{\left|2\,b^2x+\sqrt{-4\,ab+1}|b|+|b|\right|}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="giac")
```

```
[Out] -log(abs(2*b^2*x + sqrt(-4*a*b + 1)*abs(b) - abs(b))/abs(2*b^2*x + sqrt(-4*
a*b + 1)*abs(b) + abs(b)))/abs(b)
```

3.101 $\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx$

Optimal. Leaf size=17

$$\csc\left(\frac{1}{7}\right)\tan^{-1}\left(\csc\left(\frac{1}{7}\right)\left(x+\cos\left(\frac{1}{7}\right)\right)\right)$$

```
[Out] ArcTan[(x + Cos[1/7])*Csc[1/7]]*Csc[1/7]
```

Rubi [A] time = 0.0175999, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {618, 204}

$$\csc\left(\frac{1}{7}\right)\tan^{-1}\left(\csc\left(\frac{1}{7}\right)\left(x+\cos\left(\frac{1}{7}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + 2*x*Cos[1/7])^(-1),x]
```

```
[Out] ArcTan[(x + Cos[1/7])*Csc[1/7]]*Csc[1/7]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-x^2-4\sin^2\left(\frac{1}{7}\right)}\,dx, x, 2x+2\cos\left(\frac{1}{7}\right)\right)\right)\\ &= \tan^{-1}\left(\left(x+\cos\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)\end{aligned}$$

Mathematica [A] time = 0.0203081, size = 19, normalized size = 1.12

$$\csc\left(\frac{1}{7}\right)\tan^{-1}\left(\frac{(x-1)\tan\left(\frac{1}{14}\right)}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + 2*x*Cos[1/7])^(-1),x]
```

```
[Out] ArcTan[((-1 + x)*Tan[1/14])/(1 + x)]*Csc[1/7]
```

Maple [B] time = 0.064, size = 33, normalized size = 1.9

$$\frac{1}{\sqrt{1-\left(\cos\left(\frac{1}{7}\right)\right)^{2}}}\arctan\left(\frac{2x+2\cos(1/7)}{2\sqrt{1-(\cos(1/7))^{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^2+2*x*cos(1/7)),x)
```

```
[Out] 1/(1-cos(1/7)^2)^(1/2)*arctan(1/2*(2*x+2*cos(1/7))/(1-cos(1/7)^2)^(1/2))
```

Maxima [B] time = 1.79494, size = 36, normalized size = 2.12

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^{2}+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="maxima")
```

```
[Out] arctan((x + cos(1/7))/sqrt(-cos(1/7)^2 + 1))/sqrt(-cos(1/7)^2 + 1)
```

Fricas [A] time = 2.25011, size = 57, normalized size = 3.35

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sin\left(\frac{1}{7}\right)}\right)}{\sin\left(\frac{1}{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="fricas")
```

```
[Out] arctan((x + cos(1/7))/sin(1/7))/sin(1/7)
```

Sympy [C] time = 0.243859, size = 165, normalized size = 9.71

$$-\frac{i\log\left(x+\cos\left(\frac{1}{7}\right)-\frac{i}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i\cos^{2}\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}\right)}{2\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i\log\left(x+\cos\left(\frac{1}{7}\right)-\frac{i\cos^{2}\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{}{\sqrt{1-\text{co}}}\right.}{2\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**2+2*x*cos(1/7)),x)
```

```
[Out] -I*log(x + cos(1/7) - I/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)) + I*cos(1/7
)**2/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)))/(2*sqrt(1 - cos(1/7))*sqrt(co
s(1/7) + 1)) + I*log(x + cos(1/7) - I*cos(1/7)**2/(sqrt(1 - cos(1/7))*sqrt(
cos(1/7) + 1)) + I/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)))/(2*sqrt(1 - cos
(1/7))*sqrt(cos(1/7) + 1))
```

Giac [B] time = 1.40627, size = 36, normalized size = 2.12

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="giac")
```

```
[Out] arctan((x + cos(1/7))/sqrt(-cos(1/7)^2 + 1))/sqrt(-cos(1/7)^2 + 1)
```

3.102 $\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx$

Optimal. Leaf size=23

$$\csc\left(\frac{\pi}{7}\right)\tan^{-1}\left(x\csc\left(\frac{\pi}{7}\right)+\cot\left(\frac{\pi}{7}\right)\right)$$

```
[Out] ArcTan[Cot[Pi/7] + x*Csc[Pi/7]]*Csc[Pi/7]
```

Rubi [A] time = 0.0277957, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {618, 204}

$$\csc\left(\frac{\pi}{7}\right)\tan^{-1}\left(\csc\left(\frac{\pi}{7}\right)\left(x+\cos\left(\frac{\pi}{7}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + 2*x*Cos[Pi/7])^(-1),x]
```

```
[Out] ArcTan[(x + Cos[Pi/7])*Csc[Pi/7]]*Csc[Pi/7]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-x^2-4\sin^2\left(\frac{\pi}{7}\right)}\,dx, x, 2x+2\cos\left(\frac{\pi}{7}\right)\right)\right)\\ &= \tan^{-1}\left(\left(x+\cos\left(\frac{\pi}{7}\right)\right)\csc\left(\frac{\pi}{7}\right)\right)\csc\left(\frac{\pi}{7}\right)\end{aligned}$$

Mathematica [B] time = 0.0397657, size = 56, normalized size = 2.43

$$\frac{2\tan^{-1}\left(\frac{2x-(-1)^{6/7}+\sqrt[7]{-1}}{\sqrt{2-(-1)^{2/7}+(-1)^{5/7}}}\right)}{\sqrt{2-(-1)^{2/7}+(-1)^{5/7}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2 + 2*x*Cos[Pi/7])^(-1),x]
```

```
[Out] (2*ArcTan[((-1)^(1/7) - (-1)^(6/7) + 2*x)/Sqrt[2 - (-1)^(2/7) + (-1)^(5/7)]
])/Sqrt[2 - (-1)^(2/7) + (-1)^(5/7)]
```

Maple [B] time = 0.091, size = 39, normalized size = 1.7

$$\frac{1}{\sqrt{1-\left(\cos\left(\frac{\pi}{7}\right)\right)^{2}}}\arctan\left(\frac{2x+2\cos(\pi/7)}{2\sqrt{1-(\cos(\pi/7))^{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^2+2*x*cos(1/7*Pi)),x)
```

```
[Out] 1/(1-cos(1/7*Pi)^2)^(1/2)*arctan(1/2*(2*x+2*cos(1/7*Pi))/(1-cos(1/7*Pi)^2)^
(1/2))
```

Maxima [A] time = 1.71017, size = 45, normalized size = 1.96

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^{2}+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="maxima")
```

```
[Out] arctan((x + cos(1/7*pi))/sqrt(-cos(1/7*pi)^2 + 1))/sqrt(-cos(1/7*pi)^2 + 1)
```

Fricas [A] time = 2.46555, size = 69, normalized size = 3.

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\pi\right)}{\sin\left(\frac{1}{7}\pi\right)}\right)}{\sin\left(\frac{1}{7}\pi\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="fricas")
```

```
[Out] arctan((x + cos(1/7*pi))/sin(1/7*pi))/sin(1/7*pi)
```

Sympy [C] time = 0.732172, size = 70, normalized size = 3.04

$$-\frac{i\log\left(x+\cos\left(\frac{\pi}{7}\right)-\frac{i\left(2-2\cos^{2}\left(\frac{\pi}{7}\right)\right)}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)}+\frac{i\log\left(x+\cos\left(\frac{\pi}{7}\right)+\frac{i\left(2-2\cos^{2}\left(\frac{\pi}{7}\right)\right)}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**2+2*x*cos(1/7*pi)),x)
```

```
[Out] -I*log(x + cos(pi/7) - I*(2 - 2*cos(pi/7)**2)/(2*sin(pi/7)))/(2*sin(pi/7))
+ I*log(x + cos(pi/7) + I*(2 - 2*cos(pi/7)**2)/(2*sin(pi/7)))/(2*sin(pi/7))
```

Giac [A] time = 1.21056, size = 45, normalized size = 1.96

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="giac")
```

```
[Out] arctan((x + cos(1/7*pi))/sqrt(-cos(1/7*pi)^2 + 1))/sqrt(-cos(1/7*pi)^2 + 1)
```

3.103 $\int \sqrt{5-6x+9x^2}\,dx$

Optimal. Leaf size=38

$$\frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)-\frac{1}{6}(1-3x)\sqrt{9x^2-6x+5}$$

```
[Out] -((1 - 3*x)*Sqrt[5 - 6*x + 9*x^2])/6 + (2*ArcSinh[(-1 + 3*x)/2])/3
```

Rubi [A] time = 0.0113981, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 215}

$$\frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)-\frac{1}{6}(1-3x)\sqrt{9x^2-6x+5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] -((1 - 3*x)*Sqrt[5 - 6*x + 9*x^2])/6 + (2*ArcSinh[(-1 + 3*x)/2])/3
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{5-6x+9x^2}\,dx &= -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2}+2\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx\\
&= -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2}+\frac{1}{18}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{144}}}\,dx, x, -6+18x\right)\\
&= -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2}+\frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(-1+3x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0165592, size = 39, normalized size = 1.03

$$\sqrt{9x^2-6x+5}\left(\frac{x}{2}-\frac{1}{6}\right)+\frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] (-1/6 + x/2)*Sqrt[5 - 6*x + 9*x^2] + (2*ArcSinh[(-1 + 3*x)/2])/3
```

Maple [A] time = 0.049, size = 29, normalized size = 0.8

$$\frac{18x-6}{36}\sqrt{9x^2-6x+5}+\frac{2}{3}\mathrm{Arcsinh}\left(-\frac{1}{2}+\frac{3x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2-6*x+5)^(1/2),x)
```

```
[Out] 1/36*(18*x-6)*(9*x^2-6*x+5)^(1/2)+2/3*arcsinh(-1/2+3/2*x)
```

Maxima [A] time = 1.61918, size = 51, normalized size = 1.34

$$\frac{1}{2}\sqrt{9x^2-6x+5}x-\frac{1}{6}\sqrt{9x^2-6x+5}+\frac{2}{3}\operatorname{arsinh}\left(\frac{3}{2}x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-6*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 - 6*x + 5)*x - 1/6*sqrt(9*x^2 - 6*x + 5) + 2/3*arcsinh(3/2*x
 - 1/2)
```

Fricas [A] time = 2.40969, size = 109, normalized size = 2.87

$$\frac{1}{6}\sqrt{9x^2-6x+5}(3x-1)-\frac{2}{3}\log\left(-3x+\sqrt{9x^2-6x+5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-6*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5)
+ 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{9x^2-6x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2-6*x+5)**(1/2),x)
```

```
[Out] Integral(sqrt(9*x**2 - 6*x + 5), x)
```

Giac [A] time = 1.20949, size = 54, normalized size = 1.42

$$\frac{1}{6}\sqrt{9x^2-6x+5}(3x-1)-\frac{2}{3}\log\left(-3x+\sqrt{9x^2-6x+5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-6*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5)
+ 1)
```

3.104 $\int \sqrt{3-4x-4x^2}\,dx$

Optimal. Leaf size=30

$$\frac{1}{4}\sqrt{-4x^2-4x+3}(2x+1)+\sin^{-1}\left(x+\frac{1}{2}\right)$$

```
[Out] ((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 + ArcSin[1/2 + x]
```

Rubi [A] time = 0.0101856, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 216}

$$\frac{1}{4}\sqrt{-4x^2-4x+3}(2x+1)+\sin^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*x - 4*x^2],x]

[Out] ((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 + ArcSin[1/2 + x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{3-4x-4x^2}\,dx &= \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2}+2\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx\\
&= \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2}-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx,x,-4-8x\right)\\
&= \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2}+\sin^{-1}\left(\frac{1}{2}+x\right)
\end{aligned}$$

Mathematica [A] time = 0.0129401, size = 30, normalized size = 1.

$$\frac{1}{4}\sqrt{-4x^2-4x+3}(2x+1)+\sin^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*x - 4*x^2],x]
```

```
[Out] ((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 + ArcSin[1/2 + x]
```

Maple [A] time = 0.048, size = 25, normalized size = 0.8

$$-\frac{-8x-4}{16}\sqrt{-4x^2-4x+3}+\arcsin\left(x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-4*x+3)^(1/2),x)
```

```
[Out] -1/16*(-8*x-4)*(-4*x^2-4*x+3)^(1/2)+arcsin(x+1/2)
```

Maxima [A] time = 1.70079, size = 51, normalized size = 1.7

$$\frac{1}{2}\sqrt{-4x^2-4x+3}x+\frac{1}{4}\sqrt{-4x^2-4x+3}-\arcsin\left(-x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 - 4*x + 3)*x + 1/4*sqrt(-4*x^2 - 4*x + 3) - arcsin(-x - 1/2
)
```

Fricas [B] time = 1.90776, size = 134, normalized size = 4.47

$$\frac{1}{4}\sqrt{-4x^2-4x+3}(2x+1)-\arctan\left(\frac{\sqrt{-4x^2-4x+3}(2x+1)}{4x^2+4x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) - arctan(sqrt(-4*x^2 - 4*x + 3)*(2*x +
 1)/(4*x^2 + 4*x - 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-4x^2-4x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-4*x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(-4*x**2 - 4*x + 3), x)
```

Giac [A] time = 1.26053, size = 32, normalized size = 1.07

$$\frac{1}{4}\sqrt{-4x^2 - 4x + 3}(2x + 1) + \arcsin\left(x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) + arcsin(x + 1/2)
```

3.105 $\int \sqrt{-8+6x+9x^2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8}-\frac{3}{2}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

```
[Out] ((1 + 3*x)*Sqrt[-8 + 6*x + 9*x^2])/6 - (3*ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x +
 9*x^2]])/2
```

Rubi [A] time = 0.0103741, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 206}

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8}-\frac{3}{2}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] ((1 + 3*x)*Sqrt[-8 + 6*x + 9*x^2])/6 - (3*ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x +
 9*x^2]])/2
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-8+6x+9x^2}\,dx &= \frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2}-\frac{9}{2}\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx\\
&= \frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2}-9\,\text{Subst}\left(\int \frac{1}{36-x^2}\,dx,x,\frac{6+18x}{\sqrt{-8+6x+9x^2}}\right)\\
&= \frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2}-\frac{3}{2}\tanh^{-1}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0155534, size = 49, normalized size = 1.

$$\left(\frac{x}{2}+\frac{1}{6}\right)\sqrt{9x^2+6x-8}-\frac{3}{2}\log\left(\sqrt{9x^2+6x-8}+3x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] (1/6 + x/2)*Sqrt[-8 + 6*x + 9*x^2] - (3*Log[1 + 3*x + Sqrt[-8 + 6*x + 9*x^2
]])/2
```

Maple [A] time = 0.049, size = 50, normalized size = 1.

$$\frac{18x+6}{36}\sqrt{9x^2+6x-8}-\frac{\sqrt{9}}{2}\ln\left(\frac{(3+9x)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2+6*x-8)^(1/2),x)
```

```
[Out] 1/36*(18*x+6)*(9*x^2+6*x-8)^(1/2)-1/2*ln(1/9*(3+9*x)*9^(1/2)+(9*x^2+6*x-8)^
(1/2))*9^(1/2)
```

Maxima [A] time = 1.4897, size = 70, normalized size = 1.43

$$\frac{1}{2}\sqrt{9x^2+6x-8}x+\frac{1}{6}\sqrt{9x^2+6x-8}-\frac{3}{2}\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+6*x-8)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 + 6*x - 8)*x + 1/6*sqrt(9*x^2 + 6*x - 8) - 3/2*log(18*x + 6*
sqrt(9*x^2 + 6*x - 8) + 6)
```

Fricas [A] time = 2.16504, size = 109, normalized size = 2.22

$$\frac{1}{6}\sqrt{9x^2+6x-8}(3x+1)+\frac{3}{2}\log\left(-3x+\sqrt{9x^2+6x-8}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+6*x-8)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(-3*x + sqrt(9*x^2 + 6*x - 8)
- 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{9x^2+6x-8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2+6*x-8)**(1/2),x)
```

```
[Out] Integral(sqrt(9*x**2 + 6*x - 8), x)
```

Giac [A] time = 1.21474, size = 55, normalized size = 1.12

$$\frac{1}{6}\sqrt{9x^2+6x-8}(3x+1)+\frac{3}{2}\log\left(\left|-3x+\sqrt{9x^2+6x-8}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+6*x-8)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(abs(-3*x + sqrt(9*x^2 + 6*x -
 8) - 1))
```

3.106 $\int \sqrt{2+4x+3x^2}\,dx$

Optimal. Leaf size=45

$$\frac{1}{6}\sqrt{3x^2+4x+2}(3x+2)+\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

```
[Out] ((2 + 3*x)*Sqrt[2 + 4*x + 3*x^2])/6 + ArcSinh[(2 + 3*x)/Sqrt[2]]/(3*Sqrt[3]
)
```

Rubi [A] time = 0.0153222, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 215}

$$\frac{1}{6}\sqrt{3x^2+4x+2}(3x+2)+\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[2 + 4*x + 3*x^2])/6 + ArcSinh[(2 + 3*x)/Sqrt[2]]/(3*Sqrt[3]
)
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{2+4x+3x^2}\,dx &= \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2}+\frac{1}{3}\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx\\ &= \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{8}}}\,dx,x,4+6x\right)}{6\sqrt{6}}\\ &= \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2}+\frac{\sinh^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0170433, size = 46, normalized size = 1.02

$$\sqrt{3x^2+4x+2}\left(\frac{x}{2}+\frac{1}{3}\right)+\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] (1/3 + x/2)*Sqrt[2 + 4*x + 3*x^2] + ArcSinh[(2 + 3*x)/Sqrt[2]]/(3*Sqrt[3])
```

Maple [A] time = 0.044, size = 35, normalized size = 0.8

$$\frac{4+6x}{12}\sqrt{3x^2+4x+2}+\frac{\sqrt{3}}{9}\text{Arcsinh}\left(\frac{3\sqrt{2}}{2}\left(x+\frac{2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+4*x+2)^(1/2),x)
```

```
[Out] 1/12*(4+6*x)*(3*x^2+4*x+2)^(1/2)+1/9*3^(1/2)*arcsinh(3/2*2^(1/2)*(x+2/3))
```

Maxima [A] time = 1.70848, size = 62, normalized size = 1.38

$$\frac{1}{2}\sqrt{3x^2+4x+2}x+\frac{1}{9}\sqrt{3}\,\text{arsinh}\left(\frac{1}{2}\sqrt{2}(3x+2)\right)+\frac{1}{3}\sqrt{3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 4*x + 2)*x + 1/9*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2)) +
1/3*sqrt(3*x^2 + 4*x + 2)
```

Fricas [A] time = 1.96425, size = 158, normalized size = 3.51

$$\frac{1}{6}\sqrt{3x^2+4x+2}(3x+2)+\frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3x^2+4x+2}(3x+2)-9x^2-12x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) + 1/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2
+ 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{3x^2+4x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 4*x + 2), x)
```

Giac [A] time = 1.2135, size = 72, normalized size = 1.6

$$\frac{1}{6}\sqrt{3x^2+4x+2}(3x+2)-\frac{1}{9}\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x+2}\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 1/9*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x -
 sqrt(3*x^2 + 4*x + 2)) - 2)
```

3.107 $\int \sqrt{2+4x-3x^2}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{6}\sqrt{-3x^2+4x+2}(2-3x)-\frac{5\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

```
[Out] -((2 - 3*x)*Sqrt[2 + 4*x - 3*x^2])/6 - (5*ArcSin[(2 - 3*x)/Sqrt[10]])/(3*Sq
rt[3])
```

Rubi [A] time = 0.0159289, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 216}

$$-\frac{1}{6}\sqrt{-3x^2+4x+2}(2-3x)-\frac{5\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 4*x - 3*x^2],x]
```

```
[Out] -((2 - 3*x)*Sqrt[2 + 4*x - 3*x^2])/6 - (5*ArcSin[(2 - 3*x)/Sqrt[10]])/(3*Sq
rt[3])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{2+4x-3x^2}\,dx &= -\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2}+\frac{5}{3}\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx\\
&= -\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2}-\frac{1}{6}\sqrt{\frac{5}{6}}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{40}}}\,dx,x,4-6x\right)\\
&= -\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2}-\frac{5\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0192693, size = 46, normalized size = 1.02

$$\left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-3x^2+4x+2}-\frac{5\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 4*x - 3*x^2],x]
```

```
[Out] (-1/3 + x/2)*Sqrt[2 + 4*x - 3*x^2] - (5*ArcSin[(2 - 3*x)/Sqrt[10]])/(3*Sqrt
[3])
```

Maple [A] time = 0.047, size = 35, normalized size = 0.8

$$-\frac{-6\,x+4}{12}\sqrt{-3\,x^2+4\,x+2}+\frac{5\,\sqrt{3}}{9}\arcsin\left(\frac{3\,\sqrt{10}}{10}\left(x-\frac{2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+4*x+2)^(1/2),x)
```

```
[Out] -1/12*(-6*x+4)*(-3*x^2+4*x+2)^(1/2)+5/9*3^(1/2)*arcsin(3/10*10^(1/2)*(x-2/3
))
```

Maxima [A] time = 1.71788, size = 62, normalized size = 1.38

$$\frac{1}{2}\sqrt{-3\,x^2+4\,x+2}x-\frac{5}{9}\sqrt{3}\arcsin\left(-\frac{1}{10}\sqrt{10}(3\,x-2)\right)-\frac{1}{3}\sqrt{-3\,x^2+4\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 4*x + 2)*x - 5/9*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2))
 - 1/3*sqrt(-3*x^2 + 4*x + 2)
```

Fricas [A] time = 1.94372, size = 166, normalized size = 3.69

$$\frac{1}{6}\sqrt{-3\,x^2+4\,x+2}(3\,x-2)-\frac{5}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+4\,x+2}(3\,x-2)}{3\left(3\,x^2-4\,x-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) - 5/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(
-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 - 4*x - 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3x^2 + 4x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 4*x + 2), x)
```

Giac [A] time = 1.26562, size = 49, normalized size = 1.09

$$\frac{1}{6}\sqrt{-3\,x^2 + 4\,x + 2}(3\,x - 2) + \frac{5}{9}\sqrt{3}\arcsin\left(\frac{1}{10}\sqrt{10}(3\,x - 2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) + 5/9*sqrt(3)*arcsin(1/10*sqrt(10)*(3*
x - 2))
```

3.108 $\int \sqrt{2+5x+3x^2}\,dx$

Optimal. Leaf size=62

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2}-\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{24\sqrt{3}}$$

```
[Out] ((5 + 6*x)*Sqrt[2 + 5*x + 3*x^2])/12 - ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2
+ 5*x + 3*x^2])]/(24*Sqrt[3])
```

Rubi [A] time = 0.014675, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 206}

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2}-\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] ((5 + 6*x)*Sqrt[2 + 5*x + 3*x^2])/12 - ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2
+ 5*x + 3*x^2])]/(24*Sqrt[3])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{2+5x+3x^2}\,dx &= \frac{1}{12}(5+6x)\sqrt{2+5x+3x^2}-\frac{1}{24}\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx\\ &= \frac{1}{12}(5+6x)\sqrt{2+5x+3x^2}-\frac{1}{12}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{5+6x}{\sqrt{2+5x+3x^2}}\right)\\ &= \frac{1}{12}(5+6x)\sqrt{2+5x+3x^2}-\frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{2+5x+3x^2}}\right)}{24\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0279583, size = 55, normalized size = 0.89

$$\frac{1}{72}\left(6(6x+5)\sqrt{3x^2+5x+2}-\sqrt{3}\log\left(2\sqrt{9x^2+15x+6}+6x+5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] (6*(5 + 6*x)*Sqrt[2 + 5*x + 3*x^2] - Sqrt[3]*Log[5 + 6*x + 2*Sqrt[6 + 15*x
+ 9*x^2]])/72
```

Maple [A] time = 0.045, size = 50, normalized size = 0.8

$$\frac{6x+5}{12}\sqrt{3x^2+5x+2}-\frac{\sqrt{3}}{72}\ln\left(\frac{\sqrt{3}}{3}\left(\frac{5}{2}+3x\right)+\sqrt{3x^2+5x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+5*x+2)^(1/2),x)
```

```
[Out] 1/12*(6*x+5)*(3*x^2+5*x+2)^(1/2)-1/72*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2
)^(1/2))*3^(1/2)
```

Maxima [A] time = 1.73953, size = 78, normalized size = 1.26

$$\frac{1}{2}\sqrt{3x^2+5x+2}x-\frac{1}{72}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)+\frac{5}{12}\sqrt{3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 5*x + 2)*x - 1/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x +
 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x + 2)
```

Fricas [A] time = 1.9746, size = 167, normalized size = 2.69

$$\frac{1}{12}\sqrt{3x^2+5x+2}(6x+5)+\frac{1}{144}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2+5x+2}(6x+5)+72x^2+120x+49\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/144*sqrt(3)*log(-4*sqrt(3)*sqrt(3*
x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x + 49)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{3x^2+5x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 5*x + 2), x)
```

Giac [A] time = 1.25088, size = 73, normalized size = 1.18

$$\frac{1}{12}\sqrt{3x^2+5x+2}(6x+5)+\frac{1}{72}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x+2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/72*sqrt(3)*log(abs(-2*sqrt(3)*(sqr
t(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
```

3.109 $\int \sqrt{2+5x-3x^2}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{12}\sqrt{-3x^2+5x+2}(5-6x)-\frac{49\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

```
[Out] -((5 - 6*x)*Sqrt[2 + 5*x - 3*x^2])/12 - (49*ArcSin[(5 - 6*x)/7])/(24*Sqrt[3
])
```

Rubi [A] time = 0.0122895, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 216}

$$-\frac{1}{12}\sqrt{-3x^2+5x+2}(5-6x)-\frac{49\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 5*x - 3*x^2],x]
```

```
[Out] -((5 - 6*x)*Sqrt[2 + 5*x - 3*x^2])/12 - (49*ArcSin[(5 - 6*x)/7])/(24*Sqrt[3
])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{2+5x-3x^2}\,dx &= -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2}+\frac{49}{24}\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx\\
&= -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2}-\frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{49}}}\,dx,x,5-6x\right)}{24\sqrt{3}}\\
&= -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2}-\frac{49\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0193787, size = 44, normalized size = 1.02

$$\left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3x^2+5x+2}-\frac{49\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 5*x - 3*x^2],x]
```

```
[Out] (-5/12 + x/2)*Sqrt[2 + 5*x - 3*x^2] - (49*ArcSin[(5 - 6*x)/7])/(24*Sqrt[3])
```

Maple [A] time = 0.046, size = 32, normalized size = 0.7

$$\frac{49\sqrt{3}}{72}\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)-\frac{5-6x}{12}\sqrt{-3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+5*x+2)^(1/2),x)
```

```
[Out] 49/72*arcsin(-5/7+6/7*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x+2)^(1/2)
```

Maxima [A] time = 1.7193, size = 55, normalized size = 1.28

$$\frac{1}{2}\sqrt{-3x^2+5x+2}x-\frac{49}{72}\sqrt{3}\arcsin\left(-\frac{6}{7}x+\frac{5}{7}\right)-\frac{5}{12}\sqrt{-3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 5*x + 2)*x - 49/72*sqrt(3)*arcsin(-6/7*x + 5/7) - 5/12*sq
rt(-3*x^2 + 5*x + 2)
```

Fricas [A] time = 1.90065, size = 170, normalized size = 3.95

$$\frac{1}{12}\sqrt{-3x^2+5x+2}(6x-5)-\frac{49}{72}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+5x+2}(6x-5)}{6\left(3x^2-5x-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) - 49/72*sqrt(3)*arctan(1/6*sqrt(3)*sq
rt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 - 5*x - 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-3x^2+5x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 5*x + 2), x)
```

Giac [A] time = 1.29829, size = 42, normalized size = 0.98

$$\frac{1}{12}\sqrt{-3\,x^2+5\,x+2}(6\,x-5)+\frac{49}{72}\,\sqrt{3}\arcsin\left(\frac{6}{7}\,x-\frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) + 49/72*sqrt(3)*arcsin(6/7*x - 5/7)
```

3.110 $\int \sqrt{-2+4x+3x^2}\,dx$

Optimal. Leaf size=59

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{5\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

```
[Out] ((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (5*ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-
2 + 4*x + 3*x^2])])/(3*Sqrt[3])
```

Rubi [A] time = 0.0129886, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 206}

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{5\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (5*ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-
2 + 4*x + 3*x^2])])/(3*Sqrt[3])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-2+4x+3x^2}\,dx &= \frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2}-\frac{5}{3}\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx\\
&= \frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2}-\frac{10}{3}\operatorname{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{4+6x}{\sqrt{-2+4x+3x^2}}\right)\\
&= \frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2}-\frac{5\tanh^{-1}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0202719, size = 53, normalized size = 0.9

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{5\log\left(\sqrt{9x^2+12x-6}+3x+2\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (5*Log[2 + 3*x + Sqrt[-6 + 12*x + 9*
x^2]])/(3*Sqrt[3])
```

Maple [A] time = 0.052, size = 50, normalized size = 0.9

$$\frac{4+6x}{12}\sqrt{3x^2+4x-2}-\frac{5\sqrt{3}}{9}\ln\left(\frac{(2+3x)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+4*x-2)^(1/2),x)
```

```
[Out] 1/12*(4+6*x)*(3*x^2+4*x-2)^(1/2)-5/9*ln(1/3*(2+3*x)*3^(1/2)+(3*x^2+4*x-2)^(
1/2))*3^(1/2)
```

Maxima [A] time = 1.68988, size = 78, normalized size = 1.32

$$\frac{1}{2}\sqrt{3x^2+4x-2}x-\frac{5}{9}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+4x-2}+6x+4\right)+\frac{1}{3}\sqrt{3x^2+4x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 4*x - 2)*x - 5/9*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x -
2) + 6*x + 4) + 1/3*sqrt(3*x^2 + 4*x - 2)
```

Fricas [A] time = 1.85619, size = 158, normalized size = 2.68

$$\frac{1}{6}\sqrt{3x^2+4x-2}(3x+2)+\frac{5}{18}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3x^2+4x-2}(3x+2)+9x^2+12x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2
+ 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3x^2 + 4x - 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 4*x - 2), x)
```

Giac [A] time = 1.16538, size = 73, normalized size = 1.24

$$\frac{1}{6}\sqrt{3x^2 + 4x - 2}(3x + 2) + \frac{5}{9}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2 + 4x - 2}\right) - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)
*x - sqrt(3*x^2 + 4*x - 2)) - 2))
```

3.111 $\int \sqrt{-2+4x-3x^2}\,dx$

Optimal. Leaf size=59

$$\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}}-\frac{1}{6}(2-3x)\sqrt{-3x^2+4x-2}$$

```
[Out] -((2 - 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 + ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 +
 4*x - 3*x^2])]/(3*Sqrt[3])
```

Rubi [A] time = 0.0157649, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 204}

$$\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}}-\frac{1}{6}(2-3x)\sqrt{-3x^2+4x-2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] -((2 - 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 + ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 +
 4*x - 3*x^2])]/(3*Sqrt[3])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{-2+4x-3x^2}\,dx &= -\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2}-\frac{1}{3}\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx\\ &= -\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx,x,\frac{4-6x}{\sqrt{-2+4x-3x^2}}\right)\\ &= -\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2}+\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0254413, size = 54, normalized size = 0.92

$$\frac{1}{6}\sqrt{-3x^2+4x-2}(3x-2)+\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{-9x^2+12x-6}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] ((-2 + 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 + ArcTan[(2 - 3*x)/Sqrt[-6 + 12*x - 9
*x^2]]/(3*Sqrt[3])
```

Maple [A] time = 0.047, size = 46, normalized size = 0.8

$$-\frac{-6x+4}{12}\sqrt{-3x^2+4x-2}-\frac{\sqrt{3}}{9}\arctan\left(\sqrt{3}\left(x-\frac{2}{3}\right)\frac{1}{\sqrt{-3x^2+4x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+4*x-2)^(1/2),x)
```

```
[Out] -1/12*(-6*x+4)*(-3*x^2+4*x-2)^(1/2)-1/9*3^(1/2)*arctan(3^(1/2)*(x-2/3)/(-3*
x^2+4*x-2)^(1/2))
```

Maxima [C] time = 1.72406, size = 62, normalized size = 1.05

$$\frac{1}{2}\sqrt{-3x^2+4x-2}x+\frac{1}{9}i\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{2}(3x-2)\right)-\frac{1}{3}\sqrt{-3x^2+4x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 4*x - 2)*x + 1/9*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2))
 - 1/3*sqrt(-3*x^2 + 4*x - 2)
```

Fricas [C] time = 2.00885, size = 244, normalized size = 4.14

$$\frac{1}{6}\sqrt{-3x^2+4x-2}(3x-2)+\frac{1}{18}i\sqrt{3}\log\left(\frac{2i\sqrt{3}\sqrt{-3x^2+4x-2}-6x+4}{x}\right)-\frac{1}{18}i\sqrt{3}\log\left(\frac{-2i\sqrt{3}\sqrt{-3x^2+4x-2}-}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(-3*x^2 + 4*x - 2)*(3*x - 2) + 1/18*I*sqrt(3)*log((2*I*sqrt(3)*sqrt
(-3*x^2 + 4*x - 2) - 6*x + 4)/x) - 1/18*I*sqrt(3)*log((-2*I*sqrt(3)*sqrt(-3
*x^2 + 4*x - 2) - 6*x + 4)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3x^2 + 4x - 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 4*x - 2), x)
```

Giac [C] time = 1.20144, size = 49, normalized size = 0.83

$$\frac{1}{6}\sqrt{-3\,x^2 + 4\,x - 2}(3\,x - 2) + \frac{1}{9}i\,\sqrt{3}\arcsin\left(\frac{1}{2}\,\sqrt{2}(3i\,x - 2i)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(-3*x^2 + 4*x - 2)*(3*x - 2) + 1/9*I*sqrt(3)*arcsin(1/2*sqrt(2)*(3*
I*x - 2*I))
```

3.112 $\int \sqrt{-2+5x+3x^2}\,dx$

Optimal. Leaf size=62

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2}-\frac{49\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{24\sqrt{3}}$$

```
[Out] ((5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2])/12 - (49*ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sq
rt[-2 + 5*x + 3*x^2])])/(24*Sqrt[3])
```

Rubi [A] time = 0.0132514, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 206}

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2}-\frac{49\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] ((5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2])/12 - (49*ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sq
rt[-2 + 5*x + 3*x^2])])/(24*Sqrt[3])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-2+5x+3x^2}\,dx &= \frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2}-\frac{49}{24}\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx\\
&= \frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2}-\frac{49}{12}\operatorname{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{5+6x}{\sqrt{-2+5x+3x^2}}\right)\\
&= \frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2}-\frac{49\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{-2+5x+3x^2}}\right)}{24\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0269587, size = 55, normalized size = 0.89

$$\frac{1}{72}\left(6(6x+5)\sqrt{3x^2+5x-2}-49\sqrt{3}\log\left(2\sqrt{9x^2+15x-6}+6x+5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] (6*(5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2] - 49*Sqrt[3]*Log[5 + 6*x + 2*Sqrt[-6 +
15*x + 9*x^2]])/72
```

Maple [A] time = 0.043, size = 50, normalized size = 0.8

$$\frac{6\,x+5}{12}\sqrt{3\,x^2+5\,x-2}-\frac{49\,\sqrt{3}}{72}\ln\left(\frac{\sqrt{3}}{3}\left(\frac{5}{2}+3\,x\right)+\sqrt{3\,x^2+5\,x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 1/12*(6*x+5)*(3*x^2+5*x-2)^(1/2)-49/72*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x-
2)^(1/2))*3^(1/2)
```

Maxima [A] time = 1.66813, size = 78, normalized size = 1.26

$$\frac{1}{2}\,\sqrt{3\,x^2+5\,x-2}x-\frac{49}{72}\,\sqrt{3}\log\left(2\,\sqrt{3}\sqrt{3\,x^2+5\,x-2}+6\,x+5\right)+\frac{5}{12}\,\sqrt{3\,x^2+5\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 5*x - 2)*x - 49/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x
- 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x - 2)
```

Fricas [A] time = 2.04633, size = 167, normalized size = 2.69

$$\frac{1}{12}\,\sqrt{3\,x^2+5\,x-2}(6\,x+5)+\frac{49}{144}\,\sqrt{3}\log\left(-4\,\sqrt{3}\sqrt{3\,x^2+5\,x-2}(6\,x+5)+72\,x^2+120\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/144*sqrt(3)*log(-4*sqrt(3)*sqrt(3
*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{3x^2+5x-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 5*x - 2), x)
```

Giac [A] time = 1.20793, size = 73, normalized size = 1.18

$$\frac{1}{12}\sqrt{3x^2+5x-2}(6x+5)+\frac{49}{72}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x-2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/72*sqrt(3)*log(abs(-2*sqrt(3)*(sq
rt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))
```

3.113 $\int \sqrt{-2+5x-3x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{12}\sqrt{-3x^2+5x-2}(5-6x)-\frac{\sin^{-1}(5-6x)}{24\sqrt{3}}$$

```
[Out] -((5 - 6*x)*Sqrt[-2 + 5*x - 3*x^2])/12 - ArcSin[5 - 6*x]/(24*Sqrt[3])
```

Rubi [A] time = 0.0076886, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 216}

$$-\frac{1}{12}\sqrt{-3x^2+5x-2}(5-6x)-\frac{\sin^{-1}(5-6x)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] -((5 - 6*x)*Sqrt[-2 + 5*x - 3*x^2])/12 - ArcSin[5 - 6*x]/(24*Sqrt[3])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-2+5x-3x^2}\,dx &= -\frac{1}{12}(5-6x)\sqrt{-2+5x-3x^2}+\frac{1}{24}\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx\\
&= -\frac{1}{12}(5-6x)\sqrt{-2+5x-3x^2}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,5-6x\right)}{24\sqrt{3}}\\
&= -\frac{1}{12}(5-6x)\sqrt{-2+5x-3x^2}-\frac{\sin^{-1}(5-6x)}{24\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0185895, size = 40, normalized size = 1.03

$$\left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3x^2+5x-2}-\frac{\sin^{-1}(5-6x)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] (-5/12 + x/2)*Sqrt[-2 + 5*x - 3*x^2] - ArcSin[5 - 6*x]/(24*Sqrt[3])
```

Maple [A] time = 0.045, size = 32, normalized size = 0.8

$$\frac{\arcsin\left(-5+6x\right)\sqrt{3}}{72}-\frac{5-6x}{12}\sqrt{-3x^{2}+5x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 1/72*arcsin(-5+6*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x-2)^(1/2)
```

Maxima [A] time = 1.74272, size = 55, normalized size = 1.41

$$\frac{1}{2}\sqrt{-3x^{2}+5x-2}x+\frac{1}{72}\sqrt{3}\arcsin\left(6x-5\right)-\frac{5}{12}\sqrt{-3x^{2}+5x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 5*x - 2)*x + 1/72*sqrt(3)*arcsin(6*x - 5) - 5/12*sqrt(-3*
x^2 + 5*x - 2)
```

Fricas [A] time = 1.96606, size = 169, normalized size = 4.33

$$\frac{1}{12}\sqrt{-3x^{2}+5x-2}(6x-5)-\frac{1}{72}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^{2}+5x-2}(6x-5)}{6\left(3x^{2}-5x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) - 1/72*sqrt(3)*arctan(1/6*sqrt(3)*sqr
t(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 - 5*x + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-3x^{2}+5x-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 5*x - 2), x)
```

Giac [A] time = 1.22334, size = 42, normalized size = 1.08

$$\frac{1}{12}\sqrt{-3x^2+5x-2}(6x-5)+\frac{1}{72}\sqrt{3}\arcsin(6x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) + 1/72*sqrt(3)*arcsin(6*x - 5)
```

3.114 $\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)$$

```
[Out] ArcSinh[(-1 + 3*x)/2]/3
```

Rubi [A] time = 0.0070962, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 215}

$$\frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] ArcSinh[(-1 + 3*x)/2]/3
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx &= \frac{1}{36}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{144}}}\,dx, x, -6+18x\right)\\ &= \frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(-1+3x)\right)\end{aligned}$$

Mathematica [A] time = 0.0051132, size = 14, normalized size = 1.

$$\frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] ArcSinh[(-1 + 3*x)/2]/3
```

Maple [A] time = 0.045, size = 9, normalized size = 0.6

$$\frac{1}{3}\text{Arcsinh}\left(-\frac{1}{2}+\frac{3x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2-6*x+5)^(1/2),x)
```

```
[Out] 1/3*arcsinh(-1/2+3/2*x)
```

Maxima [A] time = 1.69956, size = 11, normalized size = 0.79

$$\frac{1}{3}\,\text{arsinh}\left(\frac{3}{2}\,x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsinh(3/2*x - 1/2)
```

Fricas [B] time = 2.11903, size = 59, normalized size = 4.21

$$-\frac{1}{3}\,\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*log(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{9x^2-6x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2-6*x+5)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 - 6*x + 5), x)
```

Giac [B] time = 1.27011, size = 27, normalized size = 1.93

$$-\frac{1}{3}\,\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*log(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)
```

3.115 $\int \frac{1}{\sqrt{3-4x-4x^2}} \, dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin^{-1}\left(x+\frac{1}{2}\right)$$

```
[Out] ArcSin[1/2 + x]/2
```

Rubi [A] time = 0.0067516, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$\frac{1}{2}\sin^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 4*x - 4*x^2],x]
```

```
[Out] ArcSin[1/2 + x]/2
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-4x-4x^2}} \, dx &= -\left(\frac{1}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}} \, dx, x, -4-8x\right)\right)\\ &= \frac{1}{2}\sin^{-1}\left(\frac{1}{2}+x\right)\end{aligned}$$

Mathematica [A] time = 0.0060364, size = 14, normalized size = 1.4

$$-\frac{1}{2}\sin^{-1}\left(\frac{1}{2}(-2x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 4*x - 4*x^2],x]
```

```
[Out] -ArcSin[(-1 - 2*x)/2]/2
```

Maple [A] time = 0.046, size = 7, normalized size = 0.7

$$\frac{1}{2}\arcsin\left(x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2-4*x+3)^(1/2),x)
```

```
[Out] 1/2*arcsin(x+1/2)
```

Maxima [A] time = 1.6966, size = 11, normalized size = 1.1

$$-\frac{1}{2}\arcsin\left(-x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(-x - 1/2)
```

Fricas [B] time = 1.8221, size = 88, normalized size = 8.8

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-4\,x^2-4\,x+3}(2\,x+1)}{4\,x^2+4\,x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(sqrt(-4*x^2 - 4*x + 3)*(2*x + 1)/(4*x^2 + 4*x - 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-4x^2-4x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2-4*x+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-4*x**2 - 4*x + 3), x)
```

Giac [A] time = 1.15997, size = 8, normalized size = 0.8

$$\frac{1}{2}\arcsin\left(x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(x + 1/2)
```

3.116 $\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

```
[Out] ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3
```

Rubi [A] time = 0.0058374, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{36-x^2}\,dx, x, \frac{6+18x}{\sqrt{-8+6x+9x^2}}\right)\\ &= \frac{1}{3}\tanh^{-1}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0049753, size = 24, normalized size = 0.96

$$\frac{1}{3}\log\left(\sqrt{9x^2+6x-8}+3x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] Log[1 + 3*x + Sqrt[-8 + 6*x + 9*x^2]]/3
```

Maple [A] time = 0.046, size = 30, normalized size = 1.2

$$\frac{\sqrt{9}}{9}\ln\left(\frac{(3+9x)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2+6*x-8)^(1/2),x)
```

```
[Out] 1/9*ln(1/9*(3+9*x)*9^(1/2)+(9*x^2+6*x-8)^(1/2))*9^(1/2)
```

Maxima [A] time = 1.70804, size = 30, normalized size = 1.2

$$\frac{1}{3}\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log(18*x + 6*sqrt(9*x^2 + 6*x - 8) + 6)
```

Fricas [A] time = 1.98028, size = 59, normalized size = 2.36

$$-\frac{1}{3}\log\left(-3x+\sqrt{9x^2+6x-8}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*log(-3*x + sqrt(9*x^2 + 6*x - 8) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{9x^2+6x-8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2+6*x-8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 + 6*x - 8), x)
```

Giac [A] time = 1.19575, size = 28, normalized size = 1.12

$$-\frac{1}{3}\log\left(\left|-3x+\sqrt{9x^2+6x-8}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(-3*x + sqrt(9*x^2 + 6*x - 8) - 1))
```

3.117 $\int \frac{1}{\sqrt{2+4x+3x^2}} dx$

Optimal. Leaf size=18

$$\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{3}}$$

```
[Out] ArcSinh[(2 + 3*x)/Sqrt[2]]/Sqrt[3]
```

Rubi [A] time = 0.0095382, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 215}

$$\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] ArcSinh[(2 + 3*x)/Sqrt[2]]/Sqrt[3]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{2+4x+3x^2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{8}}} dx, x, 4+6x\right)}{2\sqrt{6}} \\
&= \frac{\sinh^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.006345, size = 18, normalized size = 1.

$$\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] ArcSinh[(2 + 3*x)/Sqrt[2]]/Sqrt[3]
```

Maple [A] time = 0.048, size = 15, normalized size = 0.8

$$\frac{\sqrt{3}}{3}\mathrm{Arcsinh}\left(\frac{3\sqrt{2}}{2}\left(x+\frac{2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+4*x+2)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arcsinh(3/2*2^(1/2)*(x+2/3))
```

Maxima [A] time = 1.76543, size = 22, normalized size = 1.22

$$\frac{1}{3}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{2}(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2))
```

Fricas [B] time = 1.91206, size = 105, normalized size = 5.83

$$\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3x^2+4x+2}(3x+2)-9x^2-12x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x - 5
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3x^2+4x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 + 4*x + 2), x)
```

Giac [B] time = 1.29968, size = 45, normalized size = 2.5

$$-\frac{1}{3}\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x+2}\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 4*x + 2)) - 2)
```

3.118 $\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcSin[(2 - 3*x)/Sqrt[10]]/Sqrt[3])
```

Rubi [A] time = 0.0102533, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 4*x - 3*x^2],x]

[Out] -(ArcSin[(2 - 3*x)/Sqrt[10]]/Sqrt[3])
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{40}}}\,dx, x, 4-6x\right)}{2\sqrt{30}}\\ &= -\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0064387, size = 19, normalized size = 1.

$$-\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 4*x - 3*x^2],x]
```

```
[Out] -(ArcSin[(2 - 3*x)/Sqrt[10]]/Sqrt[3])
```

Maple [A] time = 0.051, size = 15, normalized size = 0.8

$$\frac{\sqrt{3}}{3}\arcsin\left(\frac{3\sqrt{10}}{10}\left(x-\frac{2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x+2)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arcsin(3/10*10^(1/2)*(x-2/3))
```

Maxima [A] time = 1.69021, size = 22, normalized size = 1.16

$$-\frac{1}{3}\sqrt{3}\arcsin\left(-\frac{1}{10}\sqrt{10}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2))
```

Fricas [B] time = 2.01828, size = 115, normalized size = 6.05

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+4x+2}(3x-2)}{3\left(3x^2-4x-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 - 4
*x - 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3x^2+4x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 + 4*x + 2), x)
```

Giac [A] time = 1.26662, size = 22, normalized size = 1.16

$$\frac{1}{3}\sqrt{3}\arcsin\left(\frac{1}{10}\sqrt{10}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arcsin(1/10*sqrt(10)*(3*x - 2))
```

3.119 $\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{\sqrt{3}}$$

```
[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[3]
```

Rubi [A] time = 0.0081287, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[3]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{5+6x}{\sqrt{2+5x+3x^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{2+5x+3x^2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0055392, size = 28, normalized size = 0.8

$$\frac{\log\left(2\sqrt{9x^2+15x+6}+6x+5\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] Log[5 + 6*x + 2*Sqrt[6 + 15*x + 9*x^2]]/Sqrt[3]
```

Maple [A] time = 0.047, size = 30, normalized size = 0.9

$$\frac{\sqrt{3}}{3}\ln\left(\frac{\sqrt{3}}{3}\left(\frac{5}{2}+3\,x\right)+\sqrt{3\,x^{2}+5\,x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+5*x+2)^(1/2),x)
```

```
[Out] 1/3*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

Maxima [A] time = 1.57506, size = 38, normalized size = 1.09

$$\frac{1}{3}\,\sqrt{3}\log\left(2\,\sqrt{3}\sqrt{3\,x^{2}+5\,x+2}+6\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5)
```

Fricas [A] time = 2.07287, size = 111, normalized size = 3.17

$$\frac{1}{6}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^{2}+5\,x+2}(6\,x+5)+72\,x^{2}+120\,x+49\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x
+ 49)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3x^{2}+5x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 + 5*x + 2), x)
```

<u>Giac</u> [A] time = 1.28827, size = 46, normalized size = 1.31

$$-\frac{1}{3}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x+2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*log(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
```

3.120 $\int \frac{1}{\sqrt{2+5x-3x^2}} \, dx$

Optimal. Leaf size=17

$$-\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

```
[Out] -(ArcSin[(5 - 6*x)/7]/Sqrt[3])
```

Rubi [A] time = 0.006666, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x - 3*x^2],x]
```

```
[Out] -(ArcSin[(5 - 6*x)/7]/Sqrt[3])
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{2+5x-3x^2}} \, dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{49}}} \, dx, x, 5-6x\right)}{7\sqrt{3}} \\ &= -\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0061719, size = 17, normalized size = 1.

$$-\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x - 3*x^2],x]
```

```
[Out] -(ArcSin[(5 - 6*x)/7]/Sqrt[3])
```

Maple [A] time = 0.049, size = 12, normalized size = 0.7

$$\frac{\sqrt{3}}{3}\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+5*x+2)^(1/2),x)
```

```
[Out] 1/3*arcsin(-5/7+6/7*x)*3^(1/2)
```

Maxima [A] time = 1.70508, size = 15, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arcsin\left(-\frac{6}{7}x+\frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arcsin(-6/7*x + 5/7)
```

Fricas [B] time = 1.9463, size = 115, normalized size = 6.76

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+5x+2}(6x-5)}{6\left(3x^2-5x-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 - 5
*x - 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3x^2+5x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 + 5*x + 2), x)
```

Giac [A] time = 1.3186, size = 15, normalized size = 0.88

$$\frac{1}{3}\sqrt{3}\arcsin\left(\frac{6}{7}x-\frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arcsin(6/7*x - 5/7)
```

3.121 $\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{\sqrt{3}}$$

```
[Out] ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x + 3*x^2])]/Sqrt[3]
```

Rubi [A] time = 0.0077246, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x + 3*x^2])]/Sqrt[3]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{4+6x}{\sqrt{-2+4x+3x^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0067269, size = 26, normalized size = 0.81

$$\frac{\log\left(\sqrt{9x^2+12x-6}+3x+2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] Log[2 + 3*x + Sqrt[-6 + 12*x + 9*x^2]]/Sqrt[3]
```

Maple [A] time = 0.049, size = 30, normalized size = 0.9

$$\frac{\sqrt{3}}{3}\ln\left(\frac{(2+3x)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+4*x-2)^(1/2),x)
```

```
[Out] 1/3*ln(1/3*(2+3*x)*3^(1/2)+(3*x^2+4*x-2)^(1/2))*3^(1/2)
```

Maxima [A] time = 1.74095, size = 38, normalized size = 1.19

$$\frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+4x-2}+6x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x - 2) + 6*x + 4)
```

Fricas [A] time = 1.76923, size = 104, normalized size = 3.25

$$\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\sqrt{3x^2+4x-2}(3x+2)+9x^2+12x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(sqrt(3)*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3x^2+4x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 + 4*x - 2), x)
```

Giac [A] time = 1.2625, size = 46, normalized size = 1.44

$$-\frac{1}{3}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x-2}\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 4*x - 2)) - 2))
```

3.122 $\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x - 3*x^2])]/Sqrt[3])
```

Rubi [A] time = 0.0077835, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 204}

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] -(ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x - 3*x^2])]/Sqrt[3])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, \frac{4-6x}{\sqrt{-2+4x-3x^2}}\right)\\ &= -\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0072827, size = 28, normalized size = 0.85

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{-9x^2+12x-6}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] -(ArcTan[(2 - 3*x)/Sqrt[-6 + 12*x - 9*x^2]]/Sqrt[3])
```

Maple [A] time = 0.057, size = 26, normalized size = 0.8

$$\frac{\sqrt{3}}{3}\arctan\left(\sqrt{3}\left(x-\frac{2}{3}\right)\frac{1}{\sqrt{-3x^{2}+4x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x-2)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arctan(3^(1/2)*(x-2/3)/(-3*x^2+4*x-2)^(1/2))
```

Maxima [C] time = 1.82374, size = 22, normalized size = 0.67

$$-\frac{1}{3}i\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{2}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2))
```

Fricas [C] time = 1.92105, size = 190, normalized size = 5.76

$$-\frac{1}{6}i\sqrt{3}\log\left(\frac{2i\sqrt{3}\sqrt{-3x^{2}+4x-2}-6x+4}{x}\right)+\frac{1}{6}i\sqrt{3}\log\left(\frac{-2i\sqrt{3}\sqrt{-3x^{2}+4x-2}-6x+4}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*I*sqrt(3)*log((2*I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4)/x) + 1/6*
I*sqrt(3)*log((-2*I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3x^{2}+4x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 + 4*x - 2), x)
```

Giac [C] time = 1.2423, size = 22, normalized size = 0.67

$$-\frac{1}{3}i\sqrt{3}\arcsin\left(\frac{1}{2}\sqrt{2}(3ix-2i)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*I*sqrt(3)*arcsin(1/2*sqrt(2)*(3*I*x - 2*I))
```

3.123 $\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{\sqrt{3}}$$

```
[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[-2 + 5*x + 3*x^2])]/Sqrt[3]
```

Rubi [A] time = 0.0081826, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[-2 + 5*x + 3*x^2])]/Sqrt[3]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{5+6x}{\sqrt{-2+5x+3x^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{-2+5x+3x^2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0055893, size = 28, normalized size = 0.8

$$\frac{\log\left(2\sqrt{9x^2+15x-6}+6x+5\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] Log[5 + 6*x + 2*Sqrt[-6 + 15*x + 9*x^2]]/Sqrt[3]
```

Maple [A] time = 0.051, size = 30, normalized size = 0.9

$$\frac{\sqrt{3}}{3}\ln\left(\frac{\sqrt{3}}{3}\left(\frac{5}{2}+3x\right)+\sqrt{3x^2+5x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 1/3*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x-2)^(1/2))*3^(1/2)
```

Maxima [A] time = 1.84135, size = 38, normalized size = 1.09

$$\frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x-2}+6x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x - 2) + 6*x + 5)
```

Fricas [A] time = 1.98861, size = 109, normalized size = 3.11

$$\frac{1}{6}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2+5x-2}(6x+5)+72x^2+120x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120*x
+ 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3x^2+5x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 + 5*x - 2), x)
```

Giac [A] time = 1.27302, size = 46, normalized size = 1.31

$$-\frac{1}{3}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^{2}+5x-2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*log(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))
```

3.124 $\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx$

Optimal. Leaf size=13

$$-\frac{\sin^{-1}(5-6x)}{\sqrt{3}}$$

```
[Out] -(ArcSin[5 - 6*x]/Sqrt[3])
```

Rubi [A] time = 0.0040812, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\frac{\sin^{-1}(5-6x)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] -(ArcSin[5 - 6*x]/Sqrt[3])
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 5-6x\right)}{\sqrt{3}}\\ &= -\frac{\sin^{-1}(5-6x)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0059762, size = 13, normalized size = 1.

$$-\frac{\sin^{-1}(5-6x)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] -(ArcSin[5 - 6*x]/Sqrt[3])
```

Maple [A] time = 0.057, size = 12, normalized size = 0.9

$$\frac{\arcsin\left(-5+6\,x\right)\sqrt{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 1/3*arcsin(-5+6*x)*3^(1/2)
```

Maxima [A] time = 1.76947, size = 15, normalized size = 1.15

$$\frac{1}{3}\,\sqrt{3}\arcsin\left(6\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsin(6*x - 5)
```

Fricas [B] time = 1.95196, size = 115, normalized size = 8.85

$$-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+5\,x-2}(6\,x-5)}{6\left(3\,x^2-5\,x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 - 5
*x + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3x^2+5x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 + 5*x - 2), x)
```

Giac [A] time = 1.30049, size = 15, normalized size = 1.15

$$\frac{1}{3}\,\sqrt{3}\arcsin\left(6\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arcsin(6*x - 5)
```

3.125 $\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx$

Optimal. Leaf size=22

$$\frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] ArcSinh[(b + 2*c*x)/(2*Sqrt[c])]/Sqrt[c]
```

Rubi [A] time = 0.0131654, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {619, 215}

$$\frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2],x]
```

```
[Out] ArcSinh[(b + 2*c*x)/(2*Sqrt[c])]/Sqrt[c]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4c}}}\,dx,x,b+2cx\right)}{2c}\\ &= \frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0274324, size = 22, normalized size = 1.

$$\frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2],x]
```

```
[Out] ArcSinh[(b + 2*c*x)/(2*Sqrt[c])]/Sqrt[c]
```

Maple [B] time = 0.264, size = 51, normalized size = 2.3

$$\frac{\sqrt{4}}{2}\ln\left(\frac{(4\,cx+2\,b)\,\sqrt{4}}{4}\frac{1}{\sqrt{c}}+\sqrt{\frac{b^2+4\,c}{c}+4\,bx+4\,cx^2}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x)
```

```
[Out] 1/2*ln(1/4*(4*c*x+2*b)*4^(1/2)/c^(1/2)+((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2))*4
^(1/2)/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07685, size = 320, normalized size = 14.55

$$\left[\frac{\log\left(-4\,c^2x^2-4\,bcx-b^2-(2\,cx+b)\sqrt{c}\sqrt{\frac{4\,c^2x^2+4\,bcx+b^2+4\,c}{c}}-2\,c\right)}{2\,\sqrt{c}},-\frac{\sqrt{-c}\arctan\left(\frac{(2\,cx+b)\sqrt{-c}\sqrt{\frac{4\,c^2x^2+4\,bcx+b^2+4\,c}{c}}}{4\,c^2x^2+4\,bcx+b^2+4\,c}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-4*c^2*x^2 - 4*b*c*x - b^2 - (2*c*x + b)*sqrt(c)*sqrt((4*c^2*x^2 +
 4*b*c*x + b^2 + 4*c)/c) - 2*c)/sqrt(c), -sqrt(-c)*arctan((2*c*x + b)*sqrt(
-c)*sqrt((4*c^2*x^2 + 4*b*c*x + b^2 + 4*c)/c)/(4*c^2*x^2 + 4*b*c*x + b^2 +
4*c))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int\frac{1}{\sqrt{\frac{b^2}{c}+4bx+4cx^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((b**2+4*c)/c+4*b*x+4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/sqrt(b**2/c + 4*b*x + 4*c*x**2 + 4), x)
```

Giac [B] time = 1.43829, size = 66, normalized size = 3.

$$-\frac{\log\left(\left|-\left(2\sqrt{c}x-\sqrt{4\,cx^2+4\,bx+\frac{b^2+4\,c}{c}}\right)\sqrt{c}-b\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-(2*sqrt(c)*x - sqrt(4*c*x^2 + 4*b*x + (b^2 + 4*c)/c))*sqrt(c) - b
))/sqrt(c)
```

3.126 $\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] -(ArcSin[(b - 2*c*x)/(2*Sqrt[c])]/Sqrt[c])
```

Rubi [A] time = 0.0133389, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2],x]

[Out] -(ArcSin[(b - 2*c*x)/(2*Sqrt[c])]/Sqrt[c])
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4c}}}\,dx, x, b-2cx\right)}{2c}\\ &= -\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0272743, size = 23, normalized size = 1.

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2],x]

[Out] -(ArcSin[(b - 2*c*x)/(2*Sqrt[c])]/Sqrt[c])
```

Maple [B] time = 0.247, size = 44, normalized size = 1.9

$$\arctan\left(2\sqrt{c}\left(x-1/2\,\frac{b}{c}\right)\frac{1}{\sqrt{-4\,cx^2+4\,bx-\frac{b^2-4\,c}{c}}}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x)

[Out] 1/c^(1/2)*arctan(2*c^(1/2)*(x-1/2*b/c)/(-4*c*x^2+4*b*x-(b^2-4*c)/c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.97964, size = 323, normalized size = 14.04

$$\left[-\frac{\sqrt{-c}\log\left(4\,c^2x^2-4\,bcx+b^2-(2\,cx-b)\sqrt{-c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-4\,c}{c}}-2\,c\right)}{2\,c},-\frac{\arctan\left(\frac{(2\,cx-b)\sqrt{c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-4\,c}{c}}}{4\,c^2x^2-4\,bcx+b^2-4\,c}\right)}{\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-c)*log(4*c^2*x^2 - 4*b*c*x + b^2 - (2*c*x - b)*sqrt(-c)*sqrt(-(
4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c) - 2*c)/c, -arctan((2*c*x - b)*sqrt(c)*s
qrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c)/(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)
)/sqrt(c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int\frac{1}{\sqrt{-\frac{b^2}{c}+4bx-4cx^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b**2+4*c)/c+4*b*x-4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/sqrt(-b**2/c + 4*b*x - 4*c*x**2 + 4), x)
```

Giac [B] time = 1.43435, size = 72, normalized size = 3.13

$$-\frac{\log\left(\left|\left(2\sqrt{-c}x-\sqrt{-4\,cx^2+4\,bx-\frac{b^2-4\,c}{c}}\right)\sqrt{-c}+b\right|\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs((2*sqrt(-c)*x - sqrt(-4*c*x^2 + 4*b*x - (b^2 - 4*c)/c))*sqrt(-c) +
 b))/sqrt(-c)
```

3.127 $\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] -(ArcSin[(b - 2*c*x)/Sqrt[c]]/Sqrt[c])
```

Rubi [A] time = 0.0106786, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] -(ArcSin[(b - 2*c*x)/Sqrt[c]]/Sqrt[c])
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c}}}\,dx, x, b-2cx\right)}{c}\\ &= -\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0257015, size = 20, normalized size = 1.

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] -(ArcSin[(b - 2*c*x)/Sqrt[c]]/Sqrt[c])
```

Maple [B] time = 0.242, size = 44, normalized size = 2.2

$$\arctan\left(2\sqrt{c}\left(x-1/2\,\frac{b}{c}\right)\frac{1}{\sqrt{-4\,cx^2+4\,bx-\frac{b^2-c}{c}}}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x)
```

```
[Out] 1/c^(1/2)*arctan(2*c^(1/2)*(x-1/2*b/c)/(-4*c*x^2+4*b*x-(b^2-c)/c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.18356, size = 317, normalized size = 15.85

$$\left[-\frac{\sqrt{-c}\log\left(8\,c^2x^2-8\,bcx+2\,b^2-2\,(2\,cx-b)\sqrt{-c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-c}{c}}-c\right)}{2\,c},-\frac{\arctan\left(\frac{(2\,cx-b)\sqrt{c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-c}{c}}}{4\,c^2x^2-4\,bcx+b^2-c}\right)}{\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log(8*c^2*x^2 - 8*b*c*x + 2*b^2 - 2*(2*c*x - b)*sqrt(-c)*sqr
t(-(4*c^2*x^2 - 4*b*c*x + b^2 - c)/c) - c)/c, -arctan((2*c*x - b)*sqrt(c)*s
qrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - c)/c)/(4*c^2*x^2 - 4*b*c*x + b^2 - c))/sq
rt(c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int\frac{1}{\sqrt{-\frac{b^2}{c}+4bx-4cx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b**2+c)/c+4*b*x-4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/sqrt(-b**2/c + 4*b*x - 4*c*x**2 + 1), x)
```

Giac [B] time = 1.27795, size = 72, normalized size = 3.6

$$-\frac{\log\left(\left|\left(2\sqrt{-c}x - \sqrt{-4\,cx^2 + 4\,bx - \frac{b^2-c}{c}}\right)\sqrt{-c} + b\right|\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs((2*sqrt(-c)*x - sqrt(-4*c*x^2 + 4*b*x - (b^2 - c)/c))*sqrt(-c) + b
))/sqrt(-c)
```

3.128 $\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

```
[Out] (-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

Rubi [A] time = 0.0023999, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {613}

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + x^2)^(-3/2),x]
```

```
[Out] (-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(2+3x+x^2\right)^{3/2}}\,dx = -\frac{2(3+2x)}{\sqrt{2+3x+x^2}}$$

Mathematica [A] time = 0.0055414, size = 19, normalized size = 1.

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + x^2)^(-3/2),x]
```

```
[Out] (-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

Maple [A] time = 0.049, size = 24, normalized size = 1.3

$$-2\,\frac{(2+x)(1+x)(3+2x)}{\left(x^2+3x+2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+3*x+2)^(3/2),x)
```

```
[Out] -2*(2+x)*(1+x)*(3+2*x)/(x^2+3*x+2)^(3/2)
```

Maxima [A] time = 1.17317, size = 35, normalized size = 1.84

$$-\frac{4x}{\sqrt{x^2+3x+2}}-\frac{6}{\sqrt{x^2+3x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*x/sqrt(x^2 + 3*x + 2) - 6/sqrt(x^2 + 3*x + 2)
```

Fricas [B] time = 1.87356, size = 95, normalized size = 5.

$$-\frac{2\left(2x^2+\sqrt{x^2+3x+2}(2x+3)+6x+4\right)}{x^2+3x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*x^2 + sqrt(x^2 + 3*x + 2)*(2*x + 3) + 6*x + 4)/(x^2 + 3*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(x^2+3x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+3*x+2)**(3/2),x)
```

```
[Out] Integral((x**2 + 3*x + 2)**(-3/2), x)
```

Giac [A] time = 1.19274, size = 23, normalized size = 1.21

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*x + 3)/sqrt(x^2 + 3*x + 2)
```

3.129 $\int \frac{1}{\left(27-24x+4x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=23

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

```
[Out] (3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])
```

Rubi [A] time = 0.0026535, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {613}

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

Antiderivative was successfully verified.

```
[In] Int[(27 - 24*x + 4*x^2)^(-3/2),x]
```

```
[Out] (3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(27-24x+4x^2\right)^{3/2}}\,dx = \frac{3-x}{9\sqrt{27-24x+4x^2}}$$

Mathematica [A] time = 0.0065783, size = 23, normalized size = 1.

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 - 24*x + 4*x^2)^(-3/2),x]
```

```
[Out] (3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])
```

Maple [A] time = 0.042, size = 28, normalized size = 1.2

$$-\frac{(-3+2x)(2x-9)(-3+x)}{9}\left(4x^2-24x+27\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2-24*x+27)^(3/2),x)
```

```
[Out] -1/9*(-3+2*x)*(2*x-9)*(-3+x)/(4*x^2-24*x+27)^(3/2)
```

Maxima [A] time = 1.18078, size = 41, normalized size = 1.78

$$-\frac{x}{9\sqrt{4x^2-24x+27}}+\frac{1}{3\sqrt{4x^2-24x+27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/9*x/sqrt(4*x^2 - 24*x + 27) + 1/3/sqrt(4*x^2 - 24*x + 27)
```

Fricas [B] time = 1.87474, size = 112, normalized size = 4.87

$$-\frac{4x^2+2\sqrt{4x^2-24x+27}(x-3)-24x+27}{18\left(4x^2-24x+27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/18*(4*x^2 + 2*sqrt(4*x^2 - 24*x + 27)*(x - 3) - 24*x + 27)/(4*x^2 - 24*x
 + 27)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(4x^2-24x+27\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2-24*x+27)**(3/2),x)
```

```
[Out] Integral((4*x**2 - 24*x + 27)**(-3/2), x)
```

Giac [A] time = 1.28112, size = 23, normalized size = 1.

$$-\frac{x-3}{9\sqrt{4x^2-24x+27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="giac")
```

```
[Out] -1/9*(x - 3)/sqrt(4*x^2 - 24*x + 27)
```

3.130 $\int \frac{x}{\left(5-4x-x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=23

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

```
[Out] (5 - 2*x)/(9*Sqrt[5 - 4*x - x^2])
```

Rubi [A] time = 0.0045149, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {636}

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(5 - 4*x - x^2)^(3/2),x]
```

```
[Out] (5 - 2*x)/(9*Sqrt[5 - 4*x - x^2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{\left(5-4x-x^2\right)^{3/2}}\,dx=\frac{5-2x}{9\sqrt{5-4x-x^2}}$$

Mathematica [A] time = 0.0296655, size = 23, normalized size = 1.

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(5 - 4*x - x^2)^(3/2),x]
```

```
[Out] (5 - 2*x)/(9*Sqrt[5 - 4*x - x^2])
```

Maple [A] time = 0.045, size = 26, normalized size = 1.1

$$\frac{(x+5)(-1+x)(2x-5)}{9}\left(-x^2-4x+5\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2-4*x+5)^(3/2),x)
```

```
[Out] 1/9*(x+5)*(-1+x)*(2*x-5)/(-x^2-4*x+5)^(3/2)
```

Maxima [A] time = 1.18211, size = 41, normalized size = 1.78

$$-\frac{2\,x}{9\,\sqrt{-x^2-4\,x+5}}+\frac{5}{9\,\sqrt{-x^2-4\,x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/9*x/sqrt(-x^2 - 4*x + 5) + 5/9/sqrt(-x^2 - 4*x + 5)
```

Fricas [A] time = 1.92854, size = 70, normalized size = 3.04

$$\frac{\sqrt{-x^2-4\,x+5}(2\,x-5)}{9\left(x^2+4\,x-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-\left(x-1\right)\left(x+5\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2-4*x+5)**(3/2),x)
```

```
[Out] Integral(x/(-(x - 1)*(x + 5))**(3/2), x)
```

Giac [A] time = 1.32979, size = 39, normalized size = 1.7

$$\frac{\sqrt{-x^2-4\,x+5}(2\,x-5)}{9\left(x^2+4\,x-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)
```

3.131 $\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\left(-x^2-4x+5\right)^{3/2}}$$

```
[Out] (2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*x - x^2])
```

Rubi [A] time = 0.0066557, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {614, 613}

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\left(-x^2-4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x - x^2)^(-5/2),x]
```

```
[Out] (2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*x - x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(5-4x-x^2\right)^{5/2}}\,dx &= \frac{2+x}{27\left(5-4x-x^2\right)^{3/2}}+\frac{2}{27}\int \frac{1}{\left(5-4x-x^2\right)^{3/2}}\,dx\\ &= \frac{2+x}{27\left(5-4x-x^2\right)^{3/2}}+\frac{2(2+x)}{243\sqrt{5-4x-x^2}}\end{aligned}$$

Mathematica [A] time = 0.0092703, size = 31, normalized size = 0.72

$$-\frac{(x+2)\left(2x^2+8x-19\right)}{243\left(-x^2-4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x - x^2)^(-5/2),x]
```

```
[Out] -((2 + x)*(-19 + 8*x + 2*x^2))/(243*(5 - 4*x - x^2)^(3/2))
```

Maple [A] time = 0.046, size = 36, normalized size = 0.8

$$\frac{(x+5)(-1+x)\left(2x^3+12x^2-3x-38\right)}{243}\left(-x^2-4x+5\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-4*x+5)^(5/2),x)
```

```
[Out] 1/243*(x+5)*(-1+x)*(2*x^3+12*x^2-3*x-38)/(-x^2-4*x+5)^(5/2)
```

Maxima [A] time = 1.14188, size = 80, normalized size = 1.86

$$\frac{2x}{243\sqrt{-x^2-4x+5}}+\frac{4}{243\sqrt{-x^2-4x+5}}+\frac{x}{27\left(-x^2-4x+5\right)^{\frac{3}{2}}}+\frac{2}{27\left(-x^2-4x+5\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/243*x/sqrt(-x^2 - 4*x + 5) + 4/243/sqrt(-x^2 - 4*x + 5) + 1/27*x/(-x^2 -
4*x + 5)^(3/2) + 2/27/(-x^2 - 4*x + 5)^(3/2)
```

Fricas [A] time = 2.06176, size = 123, normalized size = 2.86

$$-\frac{\left(2x^3+12x^2-3x-38\right)\sqrt{-x^2-4x+5}}{243\left(x^4+8x^3+6x^2-40x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/243*(2*x^3 + 12*x^2 - 3*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^4 + 8*x^3 + 6*x^
2 - 40*x + 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-x^2-4x+5\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-4*x+5)**(5/2),x)
```

```
[Out] Integral((-x**2 - 4*x + 5)**(-5/2), x)
```

Giac [A] time = 1.31401, size = 49, normalized size = 1.14

$$-\frac{((2(x+6)x-3)x-38)\sqrt{-x^2-4x+5}}{243(x^2+4x-5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="giac")
```

```
[Out] -1/243*((2*(x + 6)*x - 3)*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^2 + 4*x - 5)^2
```

3.132 $\int \left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=122

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

```
[Out] -((2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p
)*(a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p)))
```

Rubi [A] time = 0.0359996, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {624}

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p,x]
```

```
[Out] -((2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p
)*(a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p)))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^p\,dx=-\frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx+cx^2\right)^{1+p}{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}(1+p)}$$

Mathematica [A] time = 0.10092, size = 126, normalized size = 1.03

$$\frac{2^{p-1}\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p}(a+x(b+cx))^p\,{}_2F_1\left(-p,p+1;p+2;\frac{-b-2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^p,x]
```

```
[Out] (2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(a + x*(b + c*x))^p*Hypergeomet
ric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a
*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c])^p)
```

Maple [F] time = 1.167, size = 0, normalized size = 0.

$$\int \left(c x^2 + b x + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p,x)
```

```
[Out] int((c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^2 + b x + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c x^2 + b x + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b x + c x^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p,x)
```

```
[Out] Integral((a + b*x + c*x**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p, x)
```

3.133 $\int \left(3+4x+5x^2\right)^p \, dx$

Optimal. Leaf size=37

$$5^{-p-1}11^p(5x+2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{11}(5x+2)^2\right)$$

```
[Out] 5^(-1 - p)*11^p*(2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, -(2 + 5*x)^2/11]
```

Rubi [A] time = 0.0155656, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {619, 245}

$$5^{-p-1}11^p(5x+2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{11}(5x+2)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x + 5*x^2)^p,x]
```

```
[Out] 5^(-1 - p)*11^p*(2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, -(2 + 5*x)^2/11]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(3+4x+5x^2\right)^p \, dx &= \frac{1}{2}\left(5^{-1-p}11^p\right)\text{Subst}\left(\int \left(1+\frac{x^2}{44}\right)^p \, dx, x, 4+10x\right)\\ &= 5^{-1-p}11^p(2+5x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{11}(2+5x)^2\right)\end{aligned}$$

Mathematica [A] time = 0.0104419, size = 37, normalized size = 1.

$$5^{-p-1}11^p(5x+2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{11}(5x+2)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x + 5*x^2)^p,x]
```

```
[Out] 5^(-1 - p)*11^p*(2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, -(2 + 5*x)^2/11]
```

Maple [F] time = 3.641, size = 0, normalized size = 0.

$$\int \left(5x^2+4x+3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+4*x+3)^p,x)
```

```
[Out] int((5*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(5x^2+4x+3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+4*x+3)^p,x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(5x^2+4x+3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+4*x+3)^p,x, algorithm="fricas")
```

```
[Out] integral((5*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(5x^2+4x+3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+4*x+3)**p,x)
```

```
[Out] Integral((5*x**2 + 4*x + 3)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(5x^2+4x+3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 4*x + 3)^p, x)
```

3.134 $\int \left(3+4x+4x^2\right)^p \, dx$

Optimal. Leaf size=32

$$2^{p-1}(2x+1)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{2}(2x+1)^2\right)$$

```
[Out] 2^(-1 + p)*(1 + 2*x)*Hypergeometric2F1[1/2, -p, 3/2, -(1 + 2*x)^2/2]
```

Rubi [A] time = 0.0137135, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {619, 245}

$$2^{p-1}(2x+1)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{2}(2x+1)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x + 4*x^2)^p,x]
```

```
[Out] 2^(-1 + p)*(1 + 2*x)*Hypergeometric2F1[1/2, -p, 3/2, -(1 + 2*x)^2/2]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(3+4x+4x^2\right)^p \, dx &= 2^{-3+p}\,\text{Subst}\left(\int \left(1+\frac{x^2}{32}\right)^p \, dx, x, 4+8x\right)\\ &= 2^{-1+p}(1+2x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{2}(1+2x)^2\right)\end{aligned}$$

Mathematica [A] time = 0.0072556, size = 32, normalized size = 1.

$$2^{p-3}(8x+4)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{32}(8x+4)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x + 4*x^2)^p,x]
```

```
[Out] 2^(-3 + p)*(4 + 8*x)*Hypergeometric2F1[1/2, -p, 3/2, -(4 + 8*x)^2/32]
```

Maple [F] time = 3.586, size = 0, normalized size = 0.

$$\int \left(4 x^{2}+4 x +3\right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+4*x+3)^p,x)
```

```
[Out] int((4*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4 x^{2}+4 x +3\right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+4*x+3)^p,x, algorithm="maxima")
```

```
[Out] integrate((4*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(4 x^{2}+4 x +3\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+4*x+3)^p,x, algorithm="fricas")
```

```
[Out] integral((4*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4x^{2} + 4x + 3\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+4*x+3)**p,x)
```

```
[Out] Integral((4*x**2 + 4*x + 3)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4 x^{2}+4 x +3\right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 4*x + 3)^p, x)
```

3.135 $\int \left(3+4x+3x^2\right)^p \, dx$

Optimal. Leaf size=37

$$3^{-p-1}5^p(3x+2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{5}(3x+2)^2\right)$$

```
[Out] 3^(-1 - p)*5^p*(2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, -(2 + 3*x)^2/5]
```

Rubi [A] time = 0.0155058, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {619, 245}

$$3^{-p-1}5^p(3x+2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{5}(3x+2)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x + 3*x^2)^p,x]
```

```
[Out] 3^(-1 - p)*5^p*(2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, -(2 + 3*x)^2/5]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(3+4x+3x^2\right)^p \, dx &= \frac{1}{2}\left(3^{-1-p}5^p\right)\text{Subst}\left(\int \left(1+\frac{x^2}{20}\right)^p \, dx, x, 4+6x\right)\\ &= 3^{-1-p}5^p(2+3x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{5}(2+3x)^2\right)\end{aligned}$$

Mathematica [A] time = 0.0103257, size = 37, normalized size = 1.

$$3^{-p-1}5^p(3x+2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{5}(3x+2)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x + 3*x^2)^p,x]
```

```
[Out] 3^(-1 - p)*5^p*(2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, -(2 + 3*x)^2/5]
```

Maple [F] time = 2.441, size = 0, normalized size = 0.

$$\int \left(3 x^{2}+4 x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+4*x+3)^p,x)
```

```
[Out] int((3*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(3 x^{2}+4 x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+3)^p,x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(3 x^{2}+4 x+3\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+3)^p,x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(3x^{2}+4x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+4*x+3)**p,x)
```

```
[Out] Integral((3*x**2 + 4*x + 3)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(3 x^{2}+4 x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 4*x + 3)^p, x)
```

3.136 $\int \left(3+4x+2x^2\right)^p \, dx$

Optimal. Leaf size=21

$$(x+1)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-2(x+1)^2\right)$$

```
[Out] (1 + x)*Hypergeometric2F1[1/2, -p, 3/2, -2*(1 + x)^2]
```

Rubi [A] time = 0.0104273, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {619, 245}

$$(x+1)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-2(x+1)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x + 2*x^2)^p,x]
```

```
[Out] (1 + x)*Hypergeometric2F1[1/2, -p, 3/2, -2*(1 + x)^2]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(3+4x+2x^2\right)^p \, dx &= \frac{1}{4}\operatorname{Subst}\left(\int \left(1+\frac{x^2}{8}\right)^p \, dx, x, 4+4x\right)\\ &= (1+x)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-2(1+x)^2\right)\end{aligned}$$

Mathematica [A] time = 0.0052774, size = 21, normalized size = 1.

$$(x+1)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-2(x+1)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x + 2*x^2)^p,x]
```

```
[Out] (1 + x)*Hypergeometric2F1[1/2, -p, 3/2, -2*(1 + x)^2]
```

Maple [F] time = 3.654, size = 0, normalized size = 0.

$$\int \left(2 x^{2}+4 x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+4*x+3)^p,x)
```

```
[Out] int((2*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(2 x^{2}+4 x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+4*x+3)^p,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(2 x^{2}+4 x+3\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+4*x+3)^p,x, algorithm="fricas")
```

```
[Out] integral((2*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(2x^{2}+4x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+4*x+3)**p,x)
```

```
[Out] Integral((2*x**2 + 4*x + 3)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(2 x^{2}+4 x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((2*x^2 + 4*x + 3)^p, x)
```

3.137 $\int \left(3+4x+x^2\right)^p \, dx$

Optimal. Leaf size=54

$$-\frac{2^{2p+1}(-2x-2)^{-p-1}\left(x^2+4x+3\right)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{x+3}{2}\right)}{p+1}$$

```
[Out] -((2^(1 + 2*p)*(-2 - 2*x)^(-1 - p)*(3 + 4*x + x^2)^(1 + p)*Hypergeometric2F
1[-p, 1 + p, 2 + p, (3 + x)/2])/(1 + p))
```

Rubi [A] time = 0.0102118, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {624}

$$-\frac{2^{2p+1}(-2x-2)^{-p-1}\left(x^2+4x+3\right)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{x+3}{2}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x + x^2)^p,x]
```

```
[Out] -((2^(1 + 2*p)*(-2 - 2*x)^(-1 - p)*(3 + 4*x + x^2)^(1 + p)*Hypergeometric2F
1[-p, 1 + p, 2 + p, (3 + x)/2])/(1 + p))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int \left(3+4x+x^2\right)^p \, dx = -\frac{2^{1+2p}(-2-2x)^{-1-p}\left(3+4x+x^2\right)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{3+x}{2}\right)}{1+p}$$

Mathematica [A] time = 0.0203232, size = 48, normalized size = 0.89

$$\frac{2^p(x+1)(x+3)^{-p}\left(x^2+4x+3\right)^p\,{}_2F_1\left(-p,p+1;p+2;\frac{1}{2}(-x-1)\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x + x^2)^p,x]
```

```
[Out] (2^p*(1 + x)*(3 + 4*x + x^2)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (-1 - x)
/2])/((1 + p)*(3 + x)^p)
```

Maple [F] time = 0.841, size = 0, normalized size = 0.

$$\int \left(x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x+3)^p,x)
```

```
[Out] int((x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x+3)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^2+4x+3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x+3)^p,x, algorithm="fricas")
```

```
[Out] integral((x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x+3)**p,x)
```

```
[Out] Integral((x**2 + 4*x + 3)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2 + 4*x + 3)^p, x)
```

3.138 $\int (3+4x)^p \, dx$

Optimal. Leaf size=18

$$\frac{(4x+3)^{p+1}}{4(p+1)}$$

```
[Out] (3 + 4*x)^(1 + p)/(4*(1 + p))
```

Rubi [A] time = 0.0018257, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(4x+3)^{p+1}}{4(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x)^p,x]

[Out] (3 + 4*x)^(1 + p)/(4*(1 + p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (3+4x)^p \, dx = \frac{(3+4x)^{1+p}}{4(1+p)}$$

Mathematica [A] time = 0.0072413, size = 17, normalized size = 0.94

$$\frac{(4x+3)^{p+1}}{4p+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x)^p,x]

[Out] (3 + 4*x)^(1 + p)/(4 + 4*p)
```

Maple [A] time = 0.044, size = 17, normalized size = 0.9

$$\frac{(4x+3)^{1+p}}{4p+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+3)^p,x)
```

```
[Out] 1/4*(4*x+3)^(1+p)/(1+p)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95592, size = 47, normalized size = 2.61

$$\frac{(4x+3)^p(4x+3)}{4\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x + 3)^p*(4*x + 3)/(p + 1)
```

Sympy [A] time = 0.062529, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(4x+3)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log{(4x+3)} & \text{otherwise} \end{cases}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)**p,x)
```

```
[Out] Piecewise(((4*x + 3)**(p + 1)/(p + 1), Ne(p, -1)), (log(4*x + 3), True))/4
```

Giac [A] time = 1.28638, size = 22, normalized size = 1.22

$$\frac{(4x+3)^{p+1}}{4\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)^p,x, algorithm="giac")
```

```
[Out] 1/4*(4*x + 3)^(p + 1)/(p + 1)
```

3.139 $\int (3+4x-x^2)^p \, dx$

Optimal. Leaf size=31

$$-7^p(2-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{7}(2-x)^2\right)$$

```
[Out] -(7^p*(2 - x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - x)^2/7])
```

Rubi [A] time = 0.011873, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {619, 245}

$$-7^p(2-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{7}(2-x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x - x^2)^p,x]
```

```
[Out] -(7^p*(2 - x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - x)^2/7])
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (3+4x-x^2)^p \, dx &= -\left(\frac{1}{2}7^p \operatorname{Subst}\left(\int \left(1-\frac{x^2}{28}\right)^p \, dx, x, 4-2x\right)\right)\\ &= -7^p(2-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{7}(2-x)^2\right)\end{aligned}$$

Mathematica [A] time = 0.0065657, size = 26, normalized size = 0.84

$$7^p(x-2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{7}(x-2)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x - x^2)^p,x]
```

```
[Out] 7^p*(-2 + x)*Hypergeometric2F1[1/2, -p, 3/2, (-2 + x)^2/7]
```

Maple [F] time = 0.927, size = 0, normalized size = 0.

$$\int \left(-x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+4*x+3)^p,x)
```

```
[Out] int((-x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4*x+3)^p,x, algorithm="maxima")
```

```
[Out] integrate((-x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-x^2+4x+3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4*x+3)^p,x, algorithm="fricas")
```

```
[Out] integral((-x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+4*x+3)**p,x)
```

```
[Out] Integral((-x**2 + 4*x + 3)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4*x+3)^p,x, algorithm="giac")

[Out] integrate((-x^2 + 4*x + 3)^p, x)
```

3.140 $\int (3+4x-2x^2)^p \, dx$

Optimal. Leaf size=31

$$-5^p(1-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{2}{5}(1-x)^2\right)$$

```
[Out] -(5^p*(1 - x)*Hypergeometric2F1[1/2, -p, 3/2, (2*(1 - x)^2)/5])
```

Rubi [A] time = 0.0107076, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {619, 245}

$$-5^p(1-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{2}{5}(1-x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x - 2*x^2)^p,x]

[Out] -(5^p*(1 - x)*Hypergeometric2F1[1/2, -p, 3/2, (2*(1 - x)^2)/5])
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (3+4x-2x^2)^p \, dx &= -\left(\frac{1}{4}5^p \operatorname{Subst}\left(\int \left(1-\frac{x^2}{40}\right)^p \, dx, x, 4-4x\right)\right)\\ &= -5^p(1-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{2}{5}(1-x)^2\right)\end{aligned}$$

Mathematica [A] time = 0.0066819, size = 26, normalized size = 0.84

$$5^p(x-1)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{2}{5}(x-1)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x - 2*x^2)^p,x]

[Out] 5^p*(-1 + x)*Hypergeometric2F1[1/2, -p, 3/2, (2*(-1 + x)^2)/5]
```

Maple [F] time = 0.933, size = 0, normalized size = 0.

$$\int \left(-2 x^{2}+4 x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+4*x+3)^p,x)
```

```
[Out] int((-2*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-2 x^{2}+4 x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+4*x+3)^p,x, algorithm="maxima")
```

```
[Out] integrate((-2*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-2 x^{2}+4 x+3\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+4*x+3)^p,x, algorithm="fricas")
```

```
[Out] integral((-2*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-2x^{2}+4x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+4*x+3)**p,x)
```

```
[Out] Integral((-2*x**2 + 4*x + 3)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-2 x^{2}+4 x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+4*x+3)^p,x, algorithm="giac")

[Out] integrate((-2*x^2 + 4*x + 3)^p, x)
```

3.141 $\int \left(3+4x-3x^2\right)^p\,dx$

Optimal. Leaf size=38

$$-3^{-p-1}13^p(2-3x)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{13}(2-3x)^2\right)$$

```
[Out] -(3^(-1 - p)*13^p*(2 - 3*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 3*x)^2/13]
)
```

Rubi [A] time = 0.0131742, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {619, 245}

$$-3^{-p-1}13^p(2-3x)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{13}(2-3x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x - 3*x^2)^p,x]
```

```
[Out] -(3^(-1 - p)*13^p*(2 - 3*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 3*x)^2/13]
)
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(3+4x-3x^2\right)^p\,dx &= -\left(\frac{1}{2}\left(3^{-1-p}13^p\right)\text{Subst}\left(\int \left(1-\frac{x^2}{52}\right)^p\,dx,x,4-6x\right)\right)\\ &= -3^{-1-p}13^p(2-3x)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{13}(2-3x)^2\right)\end{aligned}$$

Mathematica [A] time = 0.0094787, size = 37, normalized size = 0.97

$$3^{-p-1}13^p(3x-2)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{13}(2-3x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x - 3*x^2)^p,x]
```

```
[Out] 3^(-1 - p)*13^p*(-2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 3*x)^2/13]
```

Maple [F] time = 0.906, size = 0, normalized size = 0.

$$\int \left(-3 x^2+4 x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+4*x+3)^p,x)
```

```
[Out] int((-3*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-3 x^2+4 x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+3)^p,x, algorithm="maxima")
```

```
[Out] integrate((-3*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-3 x^2+4 x+3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+3)^p,x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-3x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+4*x+3)**p,x)
```

```
[Out] Integral((-3*x**2 + 4*x + 3)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-3 x^2+4 x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((-3*x^2 + 4*x + 3)^p, x)
```

3.142 $\int \left(3+4x-4x^2\right)^p \, dx$

Optimal. Leaf size=35

$$-2^{2p-1}(1-2x)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{4}(1-2x)^2\right)$$

```
[Out] -(2^(-1 + 2*p)*(1 - 2*x)*Hypergeometric2F1[1/2, -p, 3/2, (1 - 2*x)^2/4])
```

Rubi [A] time = 0.0133281, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {619, 245}

$$-2^{2p-1}(1-2x)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{4}(1-2x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x - 4*x^2)^p,x]
```

```
[Out] -(2^(-1 + 2*p)*(1 - 2*x)*Hypergeometric2F1[1/2, -p, 3/2, (1 - 2*x)^2/4])
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(3+4x-4x^2\right)^p \, dx &= -\left(2^{-3+2p}\,\text{Subst}\left(\int \left(1-\frac{x^2}{64}\right)^p \, dx, x, 4-8x\right)\right)\\ &= -2^{-1+2p}(1-2x)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{4}(1-2x)^2\right)\end{aligned}$$

Mathematica [A] time = 0.0077285, size = 35, normalized size = 1.

$$-2^{2p-3}(4-8x)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{64}(4-8x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x - 4*x^2)^p,x]
```

```
[Out] -(2^(-3 + 2*p)*(4 - 8*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 8*x)^2/64])
```

Maple [F] time = 0.919, size = 0, normalized size = 0.

$$\int \left(-4 x^{2}+4 x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+4*x+3)^p,x)
```

```
[Out] int((-4*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-4 x^{2}+4 x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+4*x+3)^p,x, algorithm="maxima")
```

```
[Out] integrate((-4*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-4 x^{2}+4 x+3\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+4*x+3)^p,x, algorithm="fricas")
```

```
[Out] integral((-4*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-4x^{2}+4x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+4*x+3)**p,x)
```

```
[Out] Integral((-4*x**2 + 4*x + 3)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-4 x^{2}+4 x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((-4*x^2 + 4*x + 3)^p, x)
```

3.143 $\int \left(3+4x-5x^2\right)^p\,dx$

Optimal. Leaf size=38

$$-5^{-p-1}19^p(2-5x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{19}(2-5x)^2\right)$$

```
[Out] -(5^(-1 - p)*19^p*(2 - 5*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 5*x)^2/19]
)
```

Rubi [A] time = 0.0127279, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {619, 245}

$$-5^{-p-1}19^p(2-5x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{19}(2-5x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x - 5*x^2)^p,x]
```

```
[Out] -(5^(-1 - p)*19^p*(2 - 5*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 5*x)^2/19]
)
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(3+4x-5x^2\right)^p\,dx &= -\left(\frac{1}{2}\left(5^{-1-p}19^p\right)\text{Subst}\left(\int \left(1-\frac{x^2}{76}\right)^p\,dx,x,4-10x\right)\right)\\ &= -5^{-1-p}19^p(2-5x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{19}(2-5x)^2\right)\end{aligned}$$

Mathematica [A] time = 0.0097651, size = 37, normalized size = 0.97

$$5^{-p-1}19^p(5x-2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{19}(2-5x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x - 5*x^2)^p,x]
```

```
[Out] 5^(-1 - p)*19^p*(-2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 5*x)^2/19]
```

Maple [F] time = 0.925, size = 0, normalized size = 0.

$$\int \left(-5x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5*x^2+4*x+3)^p,x)
```

```
[Out] int((-5*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-5x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+4*x+3)^p,x, algorithm="maxima")
```

```
[Out] integrate((-5*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-5x^2+4x+3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+4*x+3)^p,x, algorithm="fricas")
```

```
[Out] integral((-5*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-5x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x**2+4*x+3)**p,x)
```

```
[Out] Integral((-5*x**2 + 4*x + 3)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-5x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((-5*x^2 + 4*x + 3)^p, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
   not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                    #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```